Divided We Fall: International Health and Trade Coordination During a Pandemic*

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Abstract

We analyze the role of international trade and health coordination in times of a pandemic by building a two-economy, two-good trade model integrated into a micro-founded SIR model of infection dynamics. Governments can adopt containment policies to suppress infection spread domestically, and levy import tariffs to prevent infection coming from abroad. The efficient, i.e., coordinated, risk-sharing arrangement dynamically adjusts both policy instruments to share infection and economic risks internationally. However, in Nash equilibrium of uncoordinated governments with national mandates, trade policies robustly feature inefficiently high tariffs that peak with the pandemic in the foreign economy. This distorts terms of trade dynamics and magnifies the welfare costs of tariff wars during a pandemic, featuring lower levels of levels of consumption and production, as well as smaller gains via diversification of infection curves across economies. Gains from international coordination decline with the intensity of international transmission but increase in productivity-dampening effects of containment policies and congestion effects in pandemic-induced mortality.

Keywords: International Trade, Tariffs, SIR model, COVID-19, Health policies, Terms of trade

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The Covid-19 Pandemic has been truly international, spreading globally through health and economic linkages between countries and regions. To analyze and understand how coordination in international trade and health determines the impact of pandemics on the global economy, we study an epidemiological model of disease dynamics embedded in a model of international trade. Our model helps understand how the outbreak of a pandemic in one country is transmitted to other countries by trade, and how national containment measures and trade policies impact the spread of the pandemic in other countries. Given that the policy response to the pandemic of 2020 has been mostly along national lines, the role and the value of international coordination in combatting pandemics is potentially of great importance.

By way of motivation, consider the stylized facts for China and the United States presented in Figure 1 for the period December 2019 to October 2020: the evolution of the pandemic (top panel); the US-China terms of trade (second panel); and the year-on-year (y-o-y) and growth in industrial production in the two countries (third panel). The pandemic peaked in China, in terms of new infections, around late February 2020, while the first wave in the US began afterwards, beginning in April. Unsurprisingly, the y-o-y change in industrial production evolved in each country in sync with the pandemic, dipping as the pandemic took grip and recovering (in the case of China) as the pandemic subsided. Significant from an international trade perspective is the observation that the terms of trade deteriorated for the country experiencing the pandemic, with the price of US exports to China sharply deteriorating relative to imports around the peak of the first wave in the US.

Is this outcome – wherein the terms of trade deteriorate for the country experiencing the pandemic – consistent with optimal health and tariff policy decisions of national governments? Is this desirable from a social efficiency standpoint? How do health and tariff policies affect each other, and in turn, the attendant health and trade outcomes, during a pandemic? What would the outcomes be if national governments were to coordinate their health and tariff policies? To answer these important questions, we provide a theoretical framework by introducing – in a model of trade – SIR-dynamics along the lines of Kermack and McKendrick (1932) for international disease transmission and undertake robust numerical simulations of the resulting high-dimensional dynamic macroeconomic equilibrium.

Our paper makes three main contributions. First, we show that under coordinated policies, there may in fact be no trade off between free trade and national health policies. Rather than curtailing trade and the ensuing contact with "tariffs," borders should stay open when another country is hit by a pandemic. In particular, by modulating trade frictions cooperatively over time, governments are able to share both the health and the economic losses intertemporally. As trade becomes more contact-intensive, such as in travel services, these risk-sharing gains becomes weaker in magnitude, but remain relevant.

Second, we analyze the lack of international cooperation explicitly by modeling national policy-making as a non-cooperative game and studying its Nash equilibria. This involves a significant additional – and as far as we know hitherto unaddressed – degree of analytical and computational complexity. Our results show that non-cooperation produces a misguided intertemporal pattern of national policies that does not internalize the positive effects of domestic containment policies on foreign countries. Instead, non-cooperation exacerbates negative externalities, both on the health and on the economic front.

Third, we show that crucial to understanding the economic and health losses from international non-coordination is the interplay between domestic containment policies and international frictions relating to trade-war incentives. A globally efficient trade policy shares *both* health and economic risks across countries and makes it possible to use national containment policies more efficiently compared to the Nash equilibrium behavior.

It has been widely noted in the recent economic literature (Eichenbaum, Rebelo and Trabandt, 2020; Brotherhood et al., 2020, and others) that if a pandemic hits an economy, local consumption and production create health externalities among its individuals justifying domestic containment policies. One of our model's key insights is that, if the pandemic peaks asynchronously in different countries, then cross-border externalities arise; in such a setting, international trade offers a dynamic risk-sharing mechanism in both health and economic terms. This risk-sharing mechanism is a result of trade helping to sustain consumption in pandemic-affected economies without aggravating its health externalities through consumption and production. In particular, this mechanism requires the less-infected country to make a short-term sacrifice in terms of both economic and health welfare, in exchange for receiving the same type of help when it experiences a pandemic breakout. However, if the countries deal with their pandemic and trade issues in a non-cooperative manner, the resulting trade war leads to both lower economic welfare, and importantly, also worse health consequences.

We show these results on the benefits of international coordination on health and trade in a dynamic two-country model with SIR dynamics. Our model has three key ingredients. First, households in each country have preferences for the consumption of goods produced in both countries. Second, consumption of goods, both foreign and domestic, leads to disease transmission. SIR dynamics are derived from a micro-founded model of international transmission through consumption. Third, in order to manage the pandemic, countries can implement policy instruments for domestic containment and international tariffs. We consider two scenarios for such policies: uncoordinated and coordinated. Uncoordinated international activity takes the form of an infinite-horizon Nash equilibrium game between governments choosing their policies driven by national mandates, while coordinated policies are the optimal solution of a global planner.

In the model, the pandemic induces households to endogenously adjust their consumption and labor provision in order to reduce the probability of getting infected. Hence, unlike agents in classic (mechanical) epidemiological models, households understand that they are in a pandemic and react rationally. However, households do not internalize health externalities on other agents. Our model features what are likely the two most important such externalities (see, e.g., Garibaldi, Moen and Pissarides (2020) for a fuller discussion) and extends them to the international context. First, self-interested infected individuals ignore the health impact of their activity on others. Second, even healthy individuals ignore the dynamic externality on other not yet infected individuals, as they risk getting infected and thus posing a risk to others in the future. While these externalities have been analyzed in the recent macro-SIR literature (see our discussion below), they also constitute a cross-border externality through international trade.

We investigate how these externalities are optimally addressed with and without international coordination. In both settings, governments impose domestic containment policies (which we model as a "dissipative tax" on domestic consumption) during the course of the domestic infection. This policy contains the spread of the pandemic, as it further discourages households from consuming goods and internalizes the health externalities. Both uncoordinated governments and optimal coordination reduce the amount of infection during the pandemic, at the expense of substantially lower consumption and production in both countries. As a result, the levels of consumption and production in each country largely track the evolution of infected cases in each country, due to both the government's containment policies and the households' endogenous responses.

In addition to the domestic containment, policies, governments can levy import tariffs as a second instrument for addressing the international dimension of the problem. Even in the absence of a pandemic, our model features a trade war. As in the literature on international trade wars and negotiations, starting with Brander and Spencer (1985) or Ossa (2014), when countries take uncoordinated (Nash) policy decisions, they impose import tariffs that are too high relative to the coordinated (social planner) case. Such tariffs lead to poor consumption levels and choices between domestic and foreign goods, resulting in a significant loss of welfare. The pandemic fundamentally alters the temporal structure of tariffs, inducing variation that is linked to the relative state of the pandemic in the two countries. Our model predicts novel tariff patterns, resulting in important welfare consequences, depending on whether they are coordinated or uncoordinated.

Consider first the uncoordinated (Nash) case. When the pandemic hits the first country, it seeks to limit transmission of the disease domestically by imposing strong containment measures on domestic consumption. These containment measures put downward pressure on its domestic price level, resulting in a a competitive disadvantage to foreign goods. Furthermore, the lower price level in the domestic country incentivize imports from the infected country and leads to an increase in the risk of infection to the foreign country. In response, the foreign country *raises* its import tariffs beyond the case without a pandemic. This weakens the infected country's output even further and limits its consumption possibilities. In equilibrium, the infected country therefore *lowers* import tariffs below the case without a pandemic, in order to encourage its domestic households to consume more foreign goods which are less conducive to infection. As a result, uncoordinated policies modulate the tariff structure in a manner that skews the terms of trade *against* the infected country's production, aggravating economic risk-sharing possibilities in the midst of a pandemic.

Figure 2 illustrates the key insight about equilibrium terms of trade in our model. The blue line displays the terms of trade with uncoordinated policies. The dashed lines signify the peak of the panedemic in each country (by assumption, country A peaks first, then country B). The terms of trade from the perspective of country A are at their worst exactly when the pandemic peaks in that country. These dynamics of terms of trade lead to a substantial loss of risk-sharing, which manifests itself in the form of a high domestic consumption bias in the infected country. As the pandemic recedes and peaks in the foreign country, their roles are reversed in this loss of risk-sharing.

Consider now the case where the two countries coordinate on a jointly optimal outcome. The pandemic modulates the structure of tariffs in this case too, but in a manner that is exactly the *opposite* of the uncoordinated case. As domestic containment measures required to reduce domestic infections aggravate production and consumption in the infected country, the planner lowers the import tariffs in the foreign country and raises the import tariffs in the infected one. The structure of these tariffs is intriguing at first pass because it encourages both countries to consume more goods produced by the more infected country and therefore raise the likelihood of infection. However, terms of trade are now skewed in favor of the infected country's goods to ameliorate its economic situation. The red line in Figure 2 presents the terms of trade in the coordinated case. In particular, the terms of trade dynamics in the coordinated case are exactly the opposite of those in the uncoordinated equilibrium. This leads to better economic risk-sharing and manifests itself in the form of more efficient home bias in each country, in particular, a home bias far lower than in the uncoordinated case.

It is worth noting that risk-sharing in this context refers to individual risk. Once national policies are determined, the disease runs its course deterministically, with aggregate transmissions determined by the Law of Large Numbers. Government policies, however, influence the laws of motion of the domestic transmissions and can shift aggregate infection rates internationally, since the economies are linked through international trade and the ensuing infections. This results in changing infection risks for the individuals in each country. A key result of our analysis is that this intertemporal economic risk-sharing also leads to sharing of health risk: the foreign country imports a part of the infections by facilitating trade with the infected country, which encourages the infected country to shift consumption towards foreign goods and therefore prevents its domestic infection rates from rising more strongly; in turn, this risk-sharing then benefits the foreign country at the peak of its own infection. In this sense, "*trade is essential to save both lives and livelihoods*" (OECD (2020)).

This implies, from a normative standpoint, that cooperation on trade in times of a pandemic can result in both superior economic and health risk-sharing outcomes across countries. Hence, there need be no tradeoff between economic and health performance in the international context. This mirrors the argument by Antràs, Redding and Rossi-Hansberg (2020) who argue that for countries with similar disease fundamentals reducing trade frictions can increase the international spread of a pandemic, but that this effect is reversed if countries have sufficiently different health conditions. This latter situation arises endogenously in our model, as the disease spreads asymmetrically between countries. In fact, while Nash equilibrium tariff policies reduce international disease transmission compared to laissez-faire policies, they still produce worse health outcomes in each country than socially optimal coordinated policies[1].

We consider three important variations of our benchmark analysis, which also serve as useful robustness checks for our simulations. First, we allow containment policies to have a dampening effect on domestic economy productivity (not just for the diseased, but for all workers). Second, we vary the intensity of international transmission between the countries to reflect heterogeneity in the nature of trade between different countries; some countries engage in more contact-intensive trade, for example, in travel and tourism services, whereas others trade predominantly in merchandise goods and information technology services which are less contact-intensive. Third, to capture the variation across countries in the development of their healthcare infrastructure, we vary the dependence of pandemic-induced mortality on the domestic infection rate; an increase in such dependence proxies for greater congestion effects due to limited availability of medical equipment, oxygen supply, and hospital or intensive-care unit (ICU) beds. We show that the gains from international coordination decline with the intensity of international transmission, but increase in productivity-dampening effects of containment policies and congestion effects in pandemic-induced mortality.

From a technical point of view, our analysis is, as far as we are aware, the first to study Nash equilibrium with fully dynamic economic and health policies. This is computationally demanding, because strategies are high-dimensional and each iteration of the best-response algorithm requires solving a dynamic macroeconomic equilibrium model. In order to get suf-

[1]In Section 4.5, we show that this crucially depends on the availability of multiple policy instruments, by restricting our model to only one policy instrument per country, their domestic containment policies. In this model, Nash equilibrium yields outcomes that are, of course, overall inferior to the coordinated outcome, but that can result in fewer infections and deaths. In fact, the lack of coordination in Nash equilibrium leads to excessive domestic containment, exactly because an instrument to coordinate international economic activity (at least implicitly) is missing. With the additional instrument of tariffs, governments can loosen domestic containment policies by sharing the health losses more efficiently.

ficiently fast convergence we therefore model economic, health, and policy interactions as parsimoniously as possible. In particular, we restrict attention to open-loop Nash equilibria (see Fudenberg and Tirole (1991) or Dockner et al. (2000)) and thus assume that governments can commit to policy paths at the beginning of the interaction. Each government must choose a two-dimensional policy in each of the 156 weeks of the pandemic and once more for the ensuing steady state. Under this assumption the strategy spaces in the game between the two governments are 314-dimensional, where the outcome generated by any strategy profile is an infinite trajectory of consumption and production decisions by different agents and of aggregate health states in both economies. Solving even for open-loop equilibria by using modifications of standard best-response algorithms can therefore test the limits of even large computing power.

From a positive standpoint, our model can help to explain why, in the real-world scenario of uncoordinated decision-making by countries, terms of trade and economic outcomes may end up being excessively dire for the infected countries. An important insight is that the purely epidemiological consideration of "closing the borders" for trade and travel to limit the spread of infections should be weighed against its implications for loss of economic risk-sharing; indeed, our model suggests that even health outcomes end up being superior with *some* coordination on trade.

Related Literature. Our paper is related to a growing literature that studies the nexus between economics and disease[2]. On a single country level, Eichenbaum, Rebelo and Trabandt (2020) embed SIR disease dynamics into a macroeconomic model and study the tradeoffs involved with simple suppression policies. In one of the few papers on the economics of disease dynamics before 2020, Greenwood et al. (2019) analyzed the dynamics of HIV in Africa and its economic consequences. Building on this work, Brotherhood et al. (2020) analyze a rich set of behavioral patterns and show the importance of heterogeneous lockdown policies for the Covid-19 environment. Alvarez, Argente and Lippi (2021) is an early paper studying the optimal lockdown policy in a single country as a planning problem in a macreconomic disease model. Foundational work on the health externalities arising from Covid-19 is, among others, Garibaldi, Moen and Pissarides (2020) and Assenza et al. (2020). Just like our paper, these early papers are mostly concerned with delaying or flattening the infection curve; modelling dynamics with several infection waves as observed in the first 18 months of the global

[2]This literature has grown impressively during the last year, and we cannot do justice to it here. See Brodeur et al. (2020) and references therein for an early overview.

pandemic of 2020/21 requires additional model ingredients, as discussed by Atkeson (2021)[3]

Our paper studies multiple countries and international trade in multiple goods, with associated domestic and trade policies to manage the pandemic. It thus relates to other recent contributions studying hetereogeneity in macroeconomic SIR dynamics, such as Acemoglu et al. (2021) who develop an SIR model with heterogeneous groups and lockdown policies, and Kaplan, Moll and Violante (2020) who integrate the SIR disease dynamics in a heterogeneous agent new Keynesian model and study the distributional consequences of different containment strategies, with a focus similar to Glover et al. (2020). Fernandez-Villaverde and Jones (2020) estimate and simulate an SIR model by using disaggregate data from various locations and have provided an impressive overview of the international evolution of the disease on their website. In a similar vein, McKibbin and Roshen (2020) and Liu, Moon and Schorfheide (2021) estimate a DSGE model and a Bayesian panel VAR, respectively in order to make global forecasts of different health-economics scenarios.

Leibovici and Santacreu (2020) studies the role of international trade in essential goods during a pandemic with a multi-country, multi-sector model. Bonadio et al. (2021) and Yildirim et al. (2021) examine the role of global supply chains' impact on GDP growth across countries, while Meier and Pinto (2020) study the specific disruption of China-US supply chains and its impact on US production in March/April 2020 in detail. Early empirical work comparing pandemic policies internationally includes Ullah and Ajala (2020), who analyze effects of testing and lockdown in 69 countries, and Noy et al. (2020) who estimate measures of exposure, vulnerability and resilience to Covid-19 across countries.

Antràs, Redding and Rossi-Hansberg (2020) also study the economics of international trade and disease transmission. The authors develop a two-country model of household interaction in equilibrium with spatial frictions that provides a microfoundation for the international spread of a disease and a gravity model of international trade. While both our paper and their paper develop microfoundations of international SIR dynamics, they differ substantially otherwise. Our key focus is on governments, strategic national policies, and international coordination. In fact, unlike us, Antràs, Redding and Rossi-Hansberg (2020) treat the key policy frictions as exogenous parameters on which they perform comparative statics. In this sense, our paper is close to Beck and Wagner (2020) who also study cooperation across countries in containment policies in a simple two-stage model. However, their stylized model leaves aside the macroeconomic dynamics at the core of our model.

[3]A number of papers have investigated different containment policies, such as Berger, Herkenhoff and Mongey (2021) on the role of testing and case-dependent quarantine, Alon et al. (2020) on age-specific lockdown policies among sets of developing and advanced economies, and Jones, Philippon and Venkateswaran (2021) on work-from-home-policies. There is a large body of work on national fiscal and macroeconomic stabilization policies in response to the pandemic, on which we build in order to simplify the policy space as much as possible, but that is too large to review here.

Our paper owes much to the literature on trade wars and negotiations in international trade (Brander and Spencer, 1985; Perroni and Whalley, 2000; Broda, Limao and Weinstein, 2008; Ossa, 2011). Most closely related is Ossa (2014), which quantitatively studies optimal tariffs that arise during a trade war and quantifies the costs of failures of coordination on trade policy. We build on this work and study how international trade policy interacts with the global shock of a pandemic. Our model generates many of the features which are present in these standard models of trade wars, while highlighting the novel interaction between trade wars, health outcomes, and international coordination of policies[4].

1 The Model

In this section we present a two-country international trade model which embeds an epidemiological model of disease dynamics. Our model has three key ingredients. First, households in each country have preferences for the consumption of goods produced in both countries. Second, consumption of foreign goods potentially leads to the transmission of disease across countries. Third, governments in each country can impose dissipative taxes on total consumption and separately tariffs on international consumption.

Specifically, we consider a global economy with two countries, $k = A, B$. Each country has households, identical competitive firms, and a government. Time is discrete, $t = 0, 1, 2, ...$

For all variables we use the following notational convention. Variables describing consumption, production, or government activity in country $k \in \{A, B\}$ have the superscript k. When discussing a single country, the superscript $-k$ denotes the other country. To simplify the presentation, superscripts in equations referring to a single country are dropped whereever possible without ambiguity.

The households in each country are defined over a continuum of unit mass. Let S_t, I_t, R_t, and D_t denote the mass of susceptible, infected, recovered and deceased people in any of the two countries. The total population of the country at any date t then is $N_t = S_t + I_t + R_t$. Individuals are infinitely lived except for deaths from the disease. We do not distinguish between individuals and households. Households within each of the three living categories are identical. S_t^{-k}, I_t^{-k}, R_t^{-k}, and D_t^{-k} are the masses of the respective groups in the other country, if we discuss activity in one country k. $h \in \{s, i, r\}$ indicates the three health types.

[4]Note that, in addition to the large literature on trade wars, our paper also connects to a recent and growing literature on the broader theme of international coordination in economic affairs. Auray, Devereux and Eyquem (2019) study the strategic interaction of governments on both the trade front and the monetary policy front. Likewise, we also study how governments compete and coordinate when their actions affect both trade and health outcomes. Relatedly, Clayton and Schaab (2020) study the international coordination in macroprudential policies, and Egorov, Mukhin et al. (2019) study the coordination of monetary policies in a world in which international prices are sticky in dollars.

1.1 Firms and Households

There are two goods $j \in \{A, B\}$, which are denoted by subscripts throughout the paper. Each period, good j is produced in country j only, by using country j labor according to the linear technology

$$y_t = z_t \left(\ell_t(s) + \phi \ell_t(i) + \ell_t(r)\right) \tag{1}$$

where $\ell_t(h) = \ell_t^k(h)$ is the amount of labor provided by employees of health status h, and $z_t = z_t^k$ is country k's productivity. Infected individuals ($h = i$) have a lower productivity, as given by $\phi < 1$. Firms act competitively, maximizing profits and taking prices as given.

The prices of the goods in both countries are p_j, $j = A, B$. When discussing a single country k, p_{-k} denotes the price of good $j \neq k$. There are no transport costs or other exogenous physical trade frictions between countries.

Households in each country provide labor and consume a basket of the two goods A and B. Suppressing the time index for simplicity, denote the per household consumption of good j by households in country k by $c_j^k = c_j^k(h)$. Households in country k consume the goods as a basket composed by the standard CES aggregator

$$q(c_k^k, c_{-k}^k) = \left(\alpha (c_k^k)^{\frac{\sigma-1}{\sigma}} + (1-\alpha)(c_{-k}^k)^{\frac{\sigma-1}{\sigma}}\right)^{\frac{\sigma}{\sigma-1}} \tag{2}$$

where c_k^k denotes consumption of the domestic good, c_{-k}^k of the foreign good, $\alpha \in (0.5, 1)$ is the home bias for domestic consumption goods, and $\sigma > 1$ the substitution elasticity between the domestic and the foreign good. These two parameters are identical in both countries in order to focus on the pure effects of disease transmission in international trade.[5]

At each time t, the representative households in any of the two countries have the following objective function, where we suppress notation for the household's health status to simplify the presentation:

$$U_t = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} \left[v(x_\tau) - \frac{1}{2}\kappa \ell_\tau^2 \right], \tag{3}$$

where $0 < \beta < 1$ is the discount rate, $x_\tau = x_\tau^k(h)$ is the composite consumption basket, $\ell_\tau = \ell_\tau^k(h)$ labor supplied, and

$$x_\tau^k(h) = q(c_{k,\tau}^k(h), c_{-k,\tau}^k(h)). \tag{4}$$

We assume for computational simplicity that the utility of consumption is of the constant-

[5]The symmetry assumption can be dispensed with. The most interesting feature of the asymmetric model is the possibility of multiple infection waves along the logic described by Antràs, Redding and Rossi-Hansberg (2020): if the wave in country A is naturally short and weak and that of country B strong, then this may lead to a second wave in country A.

relative-risk-aversion type:

$$v'(x) = x^{-\rho},\, \rho > 0, \tag{5}$$

and will conduct all our simulations with the specification $\rho = 1$. In each country k, we denote aggregate consumption of the home good by

$$H_t^k = S_t^k c_{k,t}^k(s) + I_t^k c_{k,t}^k(i) + R_t^k c_{k,t}^k(r) \tag{6}$$

and by

$$M_t^k = S_t^k c_{-k,t}^k(s) + I_t^k c_{-k,t}^k(i) + R_t^k c_{-k,t}^k(r) \tag{7}$$

that of the foreign good (“imports"). Hence, the exports of country k are M_t^{-k}.

1.2 The Disease

Like Eichenbaum, Rebelo and Trabandt (2020), Brotherhood et al. (2020) and other recent economic contributions, we augment the classic SIR model by economic activity. Different from these contributions, we do not only include domestic economic interactions, but also interactions due to international trade. In the basic SIR model following Kermack and McKendrick (1932), an infectious individual in any given area can spread the virus at the rate ηS_t (so-called “mass action incidence"), where S_t is the number of susceptibles in that area. Hence, the mass of newly infected people in that area at time t is given by $T_t = \eta S_t I_t$. Eichenbaum, Rebelo and Trabandt (2020) generalize this to transmission through consumption and work activities in a single country by splitting the individual transmission rate ηS_t into three components to obtain

$$T_t = \left[\pi_1 c_t(s) c_t(i) + \pi_2 \ell_t(s) \ell_t(i) + \pi_3\right] S_t I_t \tag{8}$$

where $c_t(h)$ and $\ell_t(h)$ are consumption and labor, respectively, by the representative consumers.

We add a simple international economic channel to this transmission mechanism, taking into account that the consumption of imports leads to cross-border contacts that are potentially contagious. Typical examples of such imports of country k would be the delivery and installation of goods and equipment in k by producers from country $j \neq k$, tourists from country k in j, or services provided by j-firms in k.

This channel builds on the following generalization of the original SIR-type models, which we describe in more detail in Section A.2 in the Appendix. Dropping the time index for convenience, suppose individuals of country k and health status h spend a fraction $\ell^k(h)$ of their time at work, a fraction $\gamma c_k^k(h)$ of their time consuming the domestic good, a fraction $\gamma c_{-k}^k(h)$ consuming the foreign good, and a fraction f out of their home for other reasons, neither consuming nor working. The assumption is that the time spent consuming is proportional to the quantity consumed. Let η denote the probability of infection through contacts per unit of time spent on a given activity.[6] When "shopping", an individual is exposed to domestic residents and foreigners. Suppose there are I^k infected domestic individuals and I^{-k} infected foreigners. Since the contact intensity for foreign and domestic consumption is likely to differ, let η^f and η^d denote the corresponding infection probabilities, respectively. Then the probability of getting infected by domestic residents, per unit of time, from consuming domestic goods is $\eta^d\gamma c_k^k(i)I^k$ and that from consuming foreign goods $\eta^d\gamma c_{-k}^k(i)I^k$. Similarly, the probability of getting infected by foreigners, per unit of time, from consuming domestic goods (which are the foreigners' foreign goods) is $\eta^f\gamma c_k^{-k}(i)I^{-k}$ and that from consuming foreign goods (which are the foreigners' domestic goods) $\eta^f\gamma c_{-k}^{-k}(i)I^{-k}$. Hence, when consuming the bundle $(c_k^k(s), c_{-k}^k(s))$, the consumer faces the probability of infection

$$\gamma c_k^k(s)\eta^d\gamma c_k^k(i)I^k + \gamma c_{-k}^k(s)\eta^d\gamma c_{-k}^k(i)I^k = \left(c_k^k(s)c_k^k(i) + c_{-k}^k(s)c_{-k}^k(i)\right)\gamma^2\eta^d I^k$$

from domestic residents, and

$$\gamma c_k^k(s)\eta^f\gamma c_k^{-k}(i)I^{-k} + \gamma c_{-k}^k(s)\eta^f\gamma c_{-k}^{-k}(i)I^{-k} = \left(c_k^k(s)c_k^{-k}(i) + c_{-k}^k(s)c_{-k}^{-k}(i)\right)\gamma^2\eta^f I^{-k}$$

from foreigners.[7]

We assume for simplicity that there are no international encounters in non-work-non-consumption situations, and we also ignore those at the workplace. In particular, the infection risk from working $\ell^k(s)$ hours is $\eta^d\ell^k(s)\ell^k(i)I^k$, and the background risk from non-work-non-consumption activity is $\eta^d f^2 I^k$, both independent of foreign infections.

Hence, a susceptible in country k who chooses $\ell^k(s)$, $c_k^k(s)$, and $c_{-k}^k(s)$ transits to the

[6] This is approximately equal to the contact rate (say φ) times the transmission probability per unit of time (say θ). Both these parameters depend on individual behavior and policy, but for tractability we take both as given. What matters for transmission is $\varphi\theta t_c$, where t_c is the duration of contacts. Later, we model policy as influencing t_c.

[7] The difference between these two expressions is mostly due to the difference in contact intensities between domestic residents and foreigners. These are related to, but different from, the difference between contact intensities of goods and services. Importantly, consumption includes tourism, which is a large component of international trade in several countries (see, e.g., Culiuc, 2014). In standard foreign trade statistics holidays abroad therefore count as the domestic purchase of a foreign consumption good. This type of import is particularly foreign contact intensive. On the other hand, imports of so-called *mode-3-services* (commercial presence) involve hardly any additional contacts with foreigners. In Section 5.2, we vary the foreign contact intensity as a comparative static to derive some conclusions on how our results and their implications apply to merchandise versus services trade.

infectious state with probability

$$\begin{aligned}
& \tau(c_k^k(s), c_{-k}^k(s), \ell^k(s)) \qquad (9) \\
= & \left[\gamma^2\left(c_k^k(s)c_k^k(i) + c_{-k}^k(s)c_{-k}^k(i)\right) + \ell^k(s)\ell^k(i) + f^2\right]\eta^d I^k \\
+ & \left[c_k^k(s)c_k^{-k}(i) + c_{-k}^k(s)c_{-k}^{-k}(i)\right]\gamma^2\eta^f I^{-k}.
\end{aligned}$$

This yields the following number of average new infections in country k:

$$\begin{aligned}
T_t^k &= \left[\pi_1\left(c_{k,t}^k(s)c_{k,t}^k(i) + c_{-k,t}^k(s)c_{-k,t}^k(i)\right) + \pi_2\ell_t^k(s)\ell_t^k(i) + \pi_3\right] I_t^k S_t^k \\
&+ \pi_4\left[c_{k,t}^k(s)c_{k,t}^{-k}(i) + c_{-k,t}^k(s)c_{-k,t}^{-k}(i)\right] I_t^{-k} S_t^k, \qquad (10)
\end{aligned}$$

where

$$\begin{aligned}
\pi_1 &= \gamma^2\eta^d, & (11)\\
\pi_2 &= \eta^d, & (12)\\
\pi_3 &= f^2\eta^d, & (13)\\
\pi_4 &= \gamma^2\eta^f. & (14)
\end{aligned}$$

As in (8), the first three terms of (10) capture infections from domestic contacts arising during consumption, work, and all other local activity, respectively. The fourth term describes infections arising from contacts with foreigners while importing or exporting.[8] This is the international disease transmission mechanism at the heart of our analysis, of which the single country case (8) is a special case obtained by setting $c_{-k}^k = 0$, for $k = A, B$.

As in standard epidemiological models, the evolution of the transmission in any country is now given by

$$\begin{aligned}
S_{t+1} &= S_t - T_t, & (15)\\
I_{t+1} &= I_t + T_t - (p_r + p_d)I_t, & (16)\\
R_{t+1} &= R_t + p_r I_t, & (17)\\
D_{t+1} &= D_t + p_d(I_t)I_t. & (18)
\end{aligned}$$

where p_r and p_d are the fractions of infected individuals that recover or die, respectively, during the period. To capture the potential crowding out of medical resources, we allow the transition

[8] In order to simplify the model and the calibration, we do not include an international spillover-term from labor, as in π_2, which would be particularly relevant for the import and export of services. We have experimented with such a model, and our results become stronger.

probability p_d to be a function of the population currently infected I_t.[9] In order to keep the computational complexity as low as possible, we assume that the death rate is a linear (affine) function of the infection rate: $p_d(I_t) = p_d(0) + \zeta I_t$, where $\zeta \geq 0$ measures the fragility of the national health system under intensive care pressure.

Note that the system (15)–(18) is deterministic, and the overall population, $N_t = S_t + I_t + R_t$, decreases by $p_d I_t$ each period. We normalize the initial population in each country to $N_1^k = 1$. As is commonly assumed in much of the epidemiological literature at the moment, we assume that recovered individuals remain in that category for sure (i.e., acquire at least temporary immunity). Importantly, by (10), the epidemiological evolution in each country depends on that of the other.

We denote the current state of the disease by

$$\Theta_t = \left(S_t^A, I_t^A, R_t^A, S_t^B, I_t^B, R_t^B\right) \tag{19}$$

and consider a situation in which initially,

$$S_1^A = 1 - \varepsilon, I_1^A = \varepsilon, R_1^A = 0, \tag{20}$$

$$S_1^B = 1, I_1^B = R_1^B = 0, \tag{21}$$

where $\varepsilon > 0$ is a small number. Hence, the pandemic begins with a small number of infections in country A and then spreads endogenously to country B.

1.3 The Role of Government

In each country, the government can impose measures to contain the spread of the pandemic. We follow the approach taken by Eichenbaum, Rebelo and Trabandt (2020) and assume that these measures act like ad valorem "containment taxes" $\mu^k = \mu_t^k$. This means that households in country k have to pay an extra $\mu^k p_j$ per unit of consumption of good j, $j = A, B$. These additional costs include the costs of safety measures, new regulatory product features, waiting times, and all other additional costs induced by policies restricting contact and economic activity. The μ^k are material or immaterial and mostly deadweight costs of consumption. Let δ_μ^k be the exogenous fraction of these costs actually received by the government. So while μ is a policy parameter, δ_μ^k is not. The fraction $1 - \delta_\mu^k$ is pure waste from a public finance perspective

[9] The role of such "congestion externalities" has been emphasized and modelled in the work on optimal containment policies, e.g. by Brotherhood et al. (2020), Kaplan, Moll and Violante (2020), Favero (2020), and Assenza et al. (2020).

and represents frictions to reduce consumption activity or make it safer in health terms.[10] The fraction δ_μ^k is collected as revenue by the government. Since we have no other taxes in our model, we let $\delta_\mu^k = .5$ in most of our simulations in order to have some scope for government expenditure and public insurance.

Moreover, the government's domestic containment tax can also affect productivity. Specifically, in one of our model extensions, we set the productivity as $z_t^k = \bar{z}(1 - \mu_t^k)$. In our baseline case, however, we abstract away from this friction and assume constant productivity, i.e., $z_t^k = \bar{z}$.

The governments may impose additional restrictive measures on foreign goods. We consider explicit import tariffs $\nu^k \geq 0$, incurred over and above the general domestic frictions generated by μ^k. In any of the two countries $k = A, B$, households then have to pay $(1+\mu^k)p_k$ per unit of consumption of the domestic good and $(1+\mu^k+\nu^k)p_{-k}$ per unit of consumption of the foreign good. For each country, we can thus simplify notation by defining the "consumer prices"

$$\widehat{p}_k = \widehat{p}_k^k = (1+\mu^k)p_k \tag{22}$$

$$\widehat{p}_{-k} = \widehat{p}_{-k}^k = (1+\mu^k+\nu^k)p_{-k} \tag{23}$$

for the domestic and foreign goods, respectively.

The government's budget in either country therefore is

$$G_t^k = \delta_\mu^k \mu^k p_{k,t} H_t^k + (\delta_\mu^k \mu^k + \nu^k) p_{-k,t} M_t^k \tag{24}$$

In order to simplify the dynamics, we again follow Eichenbaum, Rebelo and Trabandt (2020), Brotherhood et al. (2020) and others, by assuming that households do not save or borrow. Hence, the only intertemporal link of household decisions is given by health concerns, and the budget constraint of a household of type h in country k at time t is static and given by

$$\widehat{p}_{k,t} c_{k,t}(h) + \widehat{p}_{-k,t} c_{-k,t}(h) = w_t(h)\ell_t(h) + g_t(h) + v_t, \tag{25}$$

where we have dropped the superscript k for notational convenience, and $w_t(h)$ is the domestic wage, $g_t(h)$ the per household government transfer to type h households, and v_t the per household profit of the corporate sector in the country. The government's budget constraint therefore is

$$G_t^k = (1 - D_t^k) g_t^k \tag{26}$$

[10] Like most of the literature, Kaplan, Moll and Violante (2020) recognize that, factually, containment measures mostly generate costs rather than revenue, but propose, in a normative sense, to replace pure frictions by equivalent Pigouvian taxes, i.e. to make δ_μ^k a policy instrument and set it as large as possible.

where $[1 - D_t^k]$ is the size of the population at time t, determined by the disease dynamics.

To simplify, in the current simple model we ignore health dependent redistributive policies $g_t(h)$ and simply assume public transfers to be independent of health status. Government policy therefore consists in setting the domestic containment policy μ_t^k that controls overall consumption and the tariffs ν_t^k that control imports. Once these are fixed, government spending g_t is given by the government budget constraint (24) and (26). The tariff can be used to achieve the following partially conflicting goals of trade and health policy. First, of course, tariffs raise money that can be distributed directly to households. Second, as usual, tariffs manipulate the terms of trade in favor of domestic goods and thus higher domestic labor income. Third, high tariffs (or related frictions) reduce infections through foreign contacts. And fourth, tariffs can be used to influence the infection dynamics by attempting to shift production internationally to where infection rates are lower.

Since the international infection dynamic (10) is deterministic, the interaction between the two governments is an infinite-horizon, deterministic multi-stage game with observed actions (see Fudenberg and Tirole, 1991). In a single-agent framework, conditioning on the state of nature (here: the aggregate infection state) would therefore not be necessary, and every open-loop optimal path can be implemented by closed-loop strategies (i.e., strategies that depend on time t and the state) and vice versa. In a multi-agent framework, on the other hand, conditioning on the state of nature (i.e., considering Markov Nash equilibria) usually increases the set of equilibria. Here, for computational reasons, we restrict attention to open-loop strategies, i.e., strategies that only depend on time t and not on the state. Hence, governments set their policy path initially once and for all.[11] To further simplify the computation, we assume that a vaccine or other cure is known to exist in a fixed, finite time T in the future. Hence, after date T there are no more infections and the economies operate without any SIR-dynamics.[12]

As discussed, households maximize their expected discounted utility, given government policy and the evolution of the disease. Let

$$u_t^k(h_t) = v(x_t^k(h_t)) - \frac{1}{2}\kappa \ell_t^k(h_t)^2 \tag{27}$$

denote the flow utility of households of health status h_t in country k at the household's optimum, and

$$V_t^k(h_t) = \mathbb{E}_t \sum_{\tau=t}^{\infty} \beta^{\tau-t} u_\tau^k(h_\tau) \tag{28}$$

[11] Uniqueness of equilibrium is, of course, difficult to prove. We have conducted extensive computational searches for other equilibria from different starting values, but always found the single Nash equilibrium reported in Section 4.1 below.

[12] In fact, for the parametrizations we have studied, the pandemic has run its course well before T and both countries have reached herd immunity. So this restriction is not binding.

the corresponding value functions. By symmetry, we assume that the government of country k maximizes the utilitarian welfare function

$$V^k = S_1^k V_1^k(s) + I_1^k V_1^k(i) + R_1^k V_1^k(r) \tag{29}$$

Uncoordinated Policy: Without coordination, we assume that the two governments play a non-cooperative game, where each chooses open-loop policy paths as described, such as to

$$\max_{\{\mu_t^k, \nu_t^k\}_t} V^k$$

taking the other government's policy path $\{\mu_t^{-k}, \nu_t^{-k}\}_t$ as given. A Nash equilibrium consists of two policy paths that are each optimal responses to each other.

Coordinated Policy: Alternatively, we consider the benchmark of a single social planner who makes the containment and tariff decisions for both countries in order to maximize the sum of the two countries' welfare:

$$\max_{\{\mu_t^A, \nu_t^A, \mu_t^B, \nu_t^B\}_t} V^A + V^B \tag{30}$$

2 Equilibrium Analysis

Given government policy μ_t^k, ν_t^k, and g_t^k in each country, firms maximize profits and households expected utility taking prices and the economic and epidemiological constraints as given.

2.1 Firm behavior

Because of the constant-returns-to-scale structure (1), firms make zero profits in equilibrium and hire as much labor as is supplied by households. Hence, in equilibrium, dropping the country superscript k, aggregate output in each country is

$$Y_t = z_t \left(S_t \ell_t(s) + \phi I_t \ell_t(i) + R_t \ell_t(r) \right) \tag{31}$$

wages are

$$w_t(h) = \begin{cases} \overline{w}_t & \text{for } h = s, r \\ \phi \overline{w}_t & \text{for } h = i \end{cases} \tag{32}$$

$$\overline{w}_t = p_t z_t \tag{33}$$

and firm profits are $v_t = 0$.

2.2 Household behavior

Households of each country at each date t maximize expected utility U_t given by (3) subject to the budget constraint (25). Again dropping the country superscript k, they choose their levels of domestic consumption $c_{k,t} = c_{k,t}(h)$, foreign consumption $c_{-k,t} = c_{-k,t}(h)$, and labor $\ell_t = \ell_t(h)$. They know their own health status h,[13] and the current state of the disease Θ_t, given by (19).

Using (28), in recursive terms, households thus choose current labor and consumption to maximize

$$v(x_t) - \frac{1}{2}\kappa\ell_t^2 + \beta\mathbb{E}_t V_{t+1}(h_{t+1}; \Theta_{t+1}) \tag{34}$$

where the expectation operator refers to the distribution of personal health h_{t+1} next period.

Susceptible Households. For a susceptible individual there are two possible future health states - either she remains in s or she gets infected and transits to i. Given (10), there are four possibilities to get infected. First, she may get infected from local contacts while consuming (shopping, eating out, etc.). This probability is increasing with her own time spent on that activity and the total time infected domestic or foreign individuals do the same. This corresponds to the first part of the π_1-term and of the π_4-term in (10), respectively. Second, she may get infected at work with a similar logic, which corresponds to the π_2-term. Third, she may get infected in general encounters with infected people locally, not related to consumption or work, summarized by the π_3-term. Fourth, she may get infected during the consumption of goods and services abroad or coming from abroad, which is summarized by the second part of the π_1- and of the π_4-term. While the first three possibilities refer to infections from domestic households, the fourth explicitly highlights the consumption risk from imports and exports and the associated interaction with foreigners.

As discussed in Section 1.2, when choosing $(c_k^k(s), c_{-k}^k(s), \ell^k(s)) \geq 0$, and thus the consumption basket $x^k(s)$ at time t, a susceptible household will transit to the infectious state with probability $\tau(c_k^k(s), c_{-k}^k(s), \ell^k(s))$ given by (9), where $c_k^k(i), c_{-k}^k(i), c_k^{-k}(i), c_{-k}^{-k}(i), \ell^k(i)$ are the equilibrium decisions by domestic and foreign infected households. We assume that susceptible households take this probability into account when making their decision.

[13]Hence, we ignore the problem of asymptomatic or presymptomatic infections. See, for example, von Thadden (2020) for a detailed discussion.

Bringing back the time index, at time t the value function of s-households therefore is

$$
\begin{aligned}
V_t^k(s) \quad &= \max_{c_{k,t}^k(s), c_{-k,t}^k(s), \ell_t^k(s)} v(x_t^k(s)) - \frac{1}{2}\kappa \left(\ell_t^k(s)\right)^2 + \beta \left[\tau_t^k(s) V_{t+1}^k(i) + (1-\tau_t^k(s)) V_{t+1}^k(s)\right] \\
&\quad \text{subject to} \\
&\quad x_t^k(s) = q(c_{k,t}^k(s), c_{-k,t}^k(s)) \qquad (35) \\
&\quad \widehat{p}_{k,t}^k c_{k,t}^k(s) + \widehat{p}_{-k,t}^k c_{-k,t}^k(s) = \overline{w}_t^k \ell_t^k(s) + g_t^k \qquad (36)
\end{aligned}
$$

where $\tau_t^k(s) = \tau(c_{k,t}^k(s), c_{-k,t}^k(s), \ell_t^k(s))$. Here, (35) describes the household's consumption basket according to (2), and (36) is its budget constraint.

If λ_t^{ks} is the Lagrange multiplier of the budget contraint (36), the first-order conditions for the consumption of the domestic good, the consumption of the imported good, and labor are respectively given as:

$$
\begin{aligned}
x_t^k(s)^{-\rho} \frac{\partial x_t^k(s)}{\partial c_{k,t}^k(s)} + \beta \left(\pi_1 c_{k,t}^k(i) I_t^k + \pi_4 c_{k,t}^{-k}(i) I_t^{-k}\right) \left(V_{t+1}^k(i) - V_{t+1}^k(s)\right) &= \lambda_t^{ks} \widehat{p}_{k,t}^k \\
x_t^k(s)^{-\rho} \frac{\partial x_t^k(s)}{\partial c_{-k,t}^k(s)} + \beta \left(\pi_1 c_{-k,t}^k(i) I_t^k + \pi_4 c_{-k,t}^{-k}(i) I_t^{-k}\right) \left(V_{t+1}^k(i) - V_{t+1}^k(s)\right) &= \lambda_t^{ks} \widehat{p}_{-k,t}^k \\
\kappa \ell_t^k(s) - \beta \pi_2 \ell_t^k(i) I_t^k \left(V_{t+1}^k(i) - V_{t+1}^k(s)\right) &= \lambda_t^{ks} \overline{w}_t^k
\end{aligned}
$$

where the second terms in each equation reflect the fact that consuming foreign goods and services increases the chances of getting infected through contacts with foreigners. Eliminating λ_t^{ks} and simplifying yields the following two first-order conditions for the optimal choices of susceptible individuals:

$$
\begin{aligned}
& \overline{w}_t^k \left[\alpha x_t^k(s)^{\frac{1}{\sigma}-\rho} c_{k,t}^k(s)^{-\frac{1}{\sigma}} + \beta \left(\pi_1 c_{k,t}^k(i) I_t^k + \pi_4 c_{k,t}^{-k}(i) I_t^{-k}\right) \left(V_{t+1}^k(i) - V_{t+1}^k(s)\right)\right] \\
= \; & \left[\kappa \ell_t^k(s) - \beta \pi_2 \ell_t^k(i) I_t^k \left(V_{t+1}^k(i) - V_{t+1}^k(s)\right)\right] \widehat{p}_{k,t}^k \qquad (37) \\
& \overline{w}_t^k \left[(1-\alpha) x_t^k(s)^{\frac{1}{\sigma}-\rho} c_{-k,t}^k(s)^{-\frac{1}{\sigma}} + \beta \left(\pi_1 c_{-k,t}^k(i) I_t^k + \pi_4 c_{-k,t}^{-k}(i) I_t^{-k}\right) \left(V_{t+1}^k(i) - V_{t+1}^k(s)\right)\right] \\
= \; & \left[\kappa \ell_t^k(s) - \beta \pi_2 \ell_t^k(i) I_t^k \left(V_{t+1}^k(i) - V_{t+1}^k(s)\right)\right] \widehat{p}_{-k,t}^k \qquad (38)
\end{aligned}
$$

Together with the aggregation condition (35) and the budget constraint (36), (37)–(38) determine the behavior of s-individuals as a function of current prices, the state of the pandemic, the current choices of infected agents, and the policy parameters g_t^k and μ_t^k, ν_t^k (which are inherent in the consumer prices $\widehat{p}_{k,t}^k, \widehat{p}_{-k,t}^k$).

Infected Households. The behavior of infected households is simpler. Their behavior has no consequences for their future health, which is exogenously given by either recovery, with probability p_r, or death, with probability p_d.

A type i household at time t therefore chooses $(c^k_{k,t}(i), c^k_{-k,t}(i), \ell^k_t(i)) \geq 0$ such as to optimize the static decision problem

$$
\begin{aligned}
V^k_t(i) \quad &= \quad \max v(x^k_t(i)) - \frac{1}{2}\kappa\left(\ell^k_t(i)\right)^2 + \beta\left[(1-p_r-p_d)V^k_{t+1}(i) + p_r V^k_{t+1}(r) + p_d V^k_{t+1}(d)\right] \\
&\qquad \text{subject to} \\
&\quad x^k_t(i) = q(c^k_{k,t}(i), c^k_{-k,t}(i)) \qquad (39) \\
&\quad \widehat{p}^k_{k,t} c^k_{k,t}(i) + \widehat{p}^k_{-k,t} c^k_{-k,t}(i) = \phi \overline{w}^k_t \ell^k_t(i) + g^k_t \qquad (40)
\end{aligned}
$$

Note that via p_d, $V^k_t(i)$ depends on the aggregate domestic pandemic state. Letting λ^{ki}_t denote the multiplier of the budget constraint, the problem yields the following three first-order conditions

$$
\begin{aligned}
x^k_t(i)^{-\rho} \frac{\partial x^k_t(i)}{\partial c^k_{k,t}(i)} \quad &= \quad \lambda^{ki}_t \widehat{p}^k_{k,t} \\
x^k_t(i)^{-\rho} \frac{\partial x^k_t(i)}{\partial c^k_{-k,t}(i)} \quad &= \quad \lambda^{ki}_t \widehat{p}^k_{-k,t} \\
\kappa \ell^k_t(i) \quad &= \quad \lambda^{ki}_t \phi \overline{w}^k_t
\end{aligned}
$$

These conditions can be further simplified and even solved explicitly for $\rho = 1$, which we do in Appendix Section A.1. Together with the aggregation condition (39) and the budget constraint (40), they determine the behavior of i-individuals as a function of current prices and the policy parameters g^k_t, μ^k_t, and ν^k_t, as well as I^k_t.

Recovered Households. Similarly, when recovered, a type r household at time t chooses $(c^k_{k,t}(r), c^k_{-k,t}(r), \ell^k_t(r)) \geq 0$ such as to optimize the static decision problem

$$
\begin{aligned}
V^k_t(r) = \quad & \max v(x^k_t(r)) - \frac{1}{2}\kappa\left(\ell^k_t(r)\right)^2 + \beta V^k_{t+1}(r) \\
& \quad \text{subject to} \\
& x^k_t(r) = q(c^k_{k,t}(r), c^k_{-k,t}(r)) \qquad (41) \\
& \widehat{p}^k_{k,t} c^k_{k,t}(r) + \widehat{p}^k_{-k,t} c^k_{-k,t}(r) = \overline{w}^k_t \ell^k_t(r) + g^k_t(r) \qquad (42)
\end{aligned}
$$

Letting λ_t^{kr} denote the multiplier of the budget constraint, the first-order conditions are

$$\begin{aligned} x_t^k(r)^{-\rho} \frac{\partial x_t^k(r)}{\partial c_{k,t}^k(r)} &= \lambda_t^{kr} \widehat{p}_{k,t}^k \\ x_t^k(r)^{-\rho} \frac{\partial x_t^k(r)}{\partial c_{-k,t}^k(r)} &= \lambda_t^{kr} \widehat{p}_{-k,t}^k \\ \kappa \ell_t^k(r) &= \lambda_t^{kr} \overline{w}_t^k \end{aligned}$$

As before, these conditions can be further simplified and even solved explicitly for $\rho = 1$, which we do in Appendix Section A.1. Together with the aggregation condition (41) and the budget constraint (42), they determine the behavior of r-individuals as a function of current prices and the policy parameters.

2.3 The macroeconomic synthesis

Each period, the following endogenous economic variables are determined in equilibrium:

- Households: 18 variables $c_{k,t}^k(h), c_{-k,t}^k(h), \ell_t^k(h)$, for $h = s, i, r$ and $k = A, B$
- Markets: 4 variables $p_{k,t}$, $\overline{w}_t^k$ for $k = A, B$, where prices, consumer prices, and government policy are linked by (22)–(23).
- Government expenditures: 2 variables g_t^k, $k = A, B$. In the absence of health dependent transfers $g_t(h)$, fiscal policy is reduced to the balanced-budget rule (26).

As argued above, given the linear production technologies, the firm variables follow automatically from the household decisions.

The governments or the common social planner set the epidemiological policy consisting of the 4 variables μ_t^k, ν_t^k, $k = A, B$, which are exogenous from the point of view of market participants. These variables are implicit in the consumer prices $\widehat{p}_{k,t}^k, \widehat{p}_{-k,t}^k$.

Counting equations, we have

- Labor markets: 2 equations in (33)
- Households: in each country 9 equations
  - for s: (36)–(38),
  - for i: (54), (55), and (51), with $w = \phi \overline{w}_t^k$, appropriately indexed.
  - for r: (54), (55), and (51), with $w = \overline{w}_t^k$, appropriately indexed.
- Goods markets: 2 equations

$$Y_t^k = \left(1 + (1 - \delta_\mu)\mu_t^k\right) H_t^k + \left(1 + (1 - \delta_\mu)\mu_t^{-k}\right) M_t^{-k} \tag{43}$$

for $k = A, B$, where output Y_t^k is given by (31), domestic consumption H_t^k by (6) and exports M_t^{-k} by (7). The right-hand side of (43) reflects the fact that the containment measures μ^k destroy real value, as measured by δ_μ.

There are 6 value functions to be solved, $V_t^k(s), V_t^k(i), V_t^k(r)$, for $k = A, B$. As usual, we normalize the value function $V_t^k(d) = 0$, assuming that the cost of death is the lost utility of life.

To help interpret the results, we define the terms of trade as the relative price of the output of country A to that of country B, before taxes and tariffs:

$$e = \frac{p^A}{p^B} \tag{44}$$

Finally, we define the aggregate consumption in each country as the population-weighted sum of the consumption baskets of all health groups

$$X_t^k = S_t^k x^k(s) + I_t^k x^k(i) + R_t^k x^k(r) \tag{45}$$

3 Parameterization

Our parameterization builds on Eichenbaum, Rebelo and Trabandt (2020). Each period in the model is a week. To save on computational costs in our high-dimensional environment, we assume log utility from consumption, i.e., we set $\rho = 1$, because this yields simple closed-form solutions to some expressions (see Appendix Section A.1).[14] We set $\beta = .96^{(1/52)}$ such that the value of life in autarky is approximately \$10 million.[15] Furthermore, we let $\phi = .8$, such that the productivity loss for infected individuals is 20%, and we set productivity $z_t = \bar{z} = 39.835$ and $\kappa = 0.001275$ so that in the pre-pandemic steady state each person works 28 hours per week and earns $58,000$ per year, consistent with average data from the U.S. Bureau of Economic Analysis and the Bureau of Labor Statistics in 2018. Initial populations are normalized to 1. In the pre-pandemic steady state the countries are symmetric.

We follow Costinot and Rodríguez-Clare (2014) and set $\sigma = 6$. The home bias parameter $\alpha = 0.53$ is chosen such that the pre-pandemic steady-state domestic consumption share is 66%.

To fix ideas, we assume that the infection originates in country A with an initial infected population of 0.001 (0.1%). It then spreads to country B via international trade, at a speed that is endogenous to each country's policy. To parameterize our disease transmission we

[14]Noting that ρ is also the inverse of the marginal rate of intertemporal substitution. Kaplan, Moll and Violante (2020) argue that also empirically $\rho = 1$ is a reasonable assumption.
[15]See, e.g., Hall, Jones and Klenow (June 2020) for a discussion.

choose π_1, π_2, and π_3 such that in a closed economy 1/6 of transmission would occur through consumption, 1/6 of transmission through production, and the remaining 2/3 of transmission through other activities. This prominent role of exogenous, behavioral transmission, which cannot be influenced by the economic policies discussed in the present model, implies that infections indeed develop into pandemics in our model.[16] We then choose π_4 such that, without government intervention, the peak of the infection in country B occurs approximately 6 months after the peak of the infection in country A. We have experimented extensively with different values of π_4 and show the results, which are remarkably similar, in Section 5.2.

Moreover, we calibrate the benchmark transition probability p_r and $p_d(0)$ so that, when the infection rate approaches 0, the baseline mortality rate is 0.5% for the infected and it takes an average of 18 days to either recover or die from infection.[17] We consider a linear specification for the death rate as a function of the infection rate: $p_d(I_t) = p_d(0) + \zeta I_t$ where $\zeta = 0.05$ in the benchmark case. This means that the mortality rate increases approximately 2.6-fold when the infection rate I_t is 10%. Again, we provide extensive comparative statics in Section 5.2.

As noted earlier, for computational reasons we cut the disease off by assuming that a vaccine becomes available after 3 years. If in our simulations we take 2 years instead of 3, the results are qualitatively unchanged. In fact, as shown below, all our simulated time series endogenously reach steady state behavior well before the end of 3 years.

We provide further details about the computation algorithm in Appendix Section A.3.

4 Results and Interpretation

The key results of our analysis are summarized in Figures 3 to 5, which contain various graphs of the evolution of our variables over the horizon of 156 weeks during which the pandemic runs its course, unchecked by vaccination. We characterize the relations of these variables analytically in Section A.1 in the appendix and, due to the complexity of the dynamics, study the key insights quantitatively in this section.[18]

Before turning to our specific results, we note that our model features one wave per coun-

[16] A policy that makes sweeping use of curfews, quarantines, and other direct non-pharmaceutical interventions in human interactions would provide a different and largely orthogonal channel to our analysis (with unmodeled dramatic economic consequences) and can potentially suppress early outbreaks by cutting these direct contacts. Given our interest in modelling the transmission of infection waves as observed in 2020/21, such radical alternatives are not very informative, and the current model seems more appropriate.

[17] Our calibration of the case fatality rate is at the lower end of the early estimates that we are aware of (see, for example, Fernandez-Villaverde and Jones (2020) or Verity et al. (2020)). These early estimates reflect high uncertainty, but also lack of experience with the treatment of severe cases.

[18] To ensure accuracy of our numerical solution we have studied how the solution varies for small permutations of the key parameters and found it to be smooth.

try.[19] The results are therefore best interpreted with respect to the first wave, which occurs in country A. This is a situation where an infection wave builds up in one country and will spread into the other one, when both countries are still far from herd immunity. Policies of both countries will therefore influence the course of the disease and its spreading in both countries. This is different in the second wave, which occurs in country B and does so at a time when country A has almost reached herd immunity and is therefore much better able to step up its production capacity to support country B. Hence, different from the global experience of the pandemic in 2021/22, where infection waves occured and ebbed repeatedly before herd immunity could be reached, the international coordination problem is of less relevance in the second wave of our model, where there is “no pandemic future".

4.1 Health and Economic Outcomes with No Government Policy

As a benchmark, Figure 3 illustrates the SIR dynamics and economic outcomes when there are no containment policies or tariffs. The top 4 panels present the disease dynamics in both countries. Starting with an initial infection rate of $I_0 = 0.001$ in country A, the pandemic quickly takes off in country A and slowly spreads to country B, where it begins to take off after around week 25. The share of infected households in country A peaks at 4.6% in week 33 and declines thereafter. Around week 50, infections in country B overtake those in A and peak at 5.5% in week 60. After week 97 the disease has run its course in country A, and after week 122 in country B, when almost 50 per cent of the population in each country has become infected and around 0.45% of the population in each country has died.

The economic outcomes track local infection rates closely. When the first wave of infection hits country A, its labor and therefore output decline by more than 15 per cent, while the values for country B stay constant (second row, fourth panel). Similarly for country B, when the pandemic hits there. In both countries during their peaks, i.e., when the domestic infection rates are either much higher or much lower than the foreign ones, domestic households increase foreign consumption. This is to reduce the exposure to domestic infection or to profit from foreigners not wanting to consume their home production. These shifts in consumption shares have a small impact on the terms of trade expressed by the relative prices of both goods (which change by at most 2 per cent).

4.2 Fully Coordinated Government Policies

Next, we consider the optimal policy by a coordinated planner who maximizes the sum of the welfare of both countries' households as given by (30). At time 0, this planner determines

[19] As discussed by Atkeson (2021), in SIR models without exogenous shocks and delays, optimal policies generally do not generate multiple waves in the propagation of the pandemic.

both countries' domestic containment policies and tariffs from week 1 to 156 until the vaccine arrives.

Figure 4 reports these internationally optimal outcomes for the respective health and economic variables. As in the no-policy case the first 4 panels show that the pandemic quickly takes off in country A and slowly spreads to country B, where it begins to take off after around week 25. The infection in country A peaks in week 33, the same time as in the unfettered outbreak, and declines thereafter. But the peak is almost 20 per cent lower and the disease lasts 9 weeks longer (see Table 2). Hence, the planner "flattens the curve".

Unlike in the no-policy case, the planner lets the pandemic hit country B less severely than country A. The infection peak in B is lower, and in B eventually 46.7 per cent of the population become infected against 49.5 per cent in A. Also A's death rate is higher. The reason is the "last mover advantage" of country B discussed above: at the time of B's infection peak, country A has almost reached herd immunity, and so the world is a less infectious place. This directly benefits B, although the planner does reduce infections in A significantly too, by shifting some to B earlier in the pandemic. This is a further expression of the asymmetry of health outcomes due to the asymmetry of the pandemic situation discussed in the introduction.

The economic outcomes react both to the infection rates and the domestic containment and tariff policies. When the first wave of infection hits country A, its labor and therefore output decline by more than 18 per cent, more than under laissez-faire (second row, fourth panel). Differently from the laissez-faire case, labor and production in B increase slightly (second row, fourth panel). Only when the second wave of infection hits country B, its consumption and labor decline significantly. The decline in both consumption and labor is much more drastic than in the laissez-faire case, which reflects the planner's tradeoff between economic welfare and health outcomes.

The planner achieves these health and economic outcomes with a combination of domestic containment measures and tariffs (second row, first and second panels). The severity of domestic containment measures in each country roughly tracks the level of infection rates in the country. On the other hand, tariffs have a different pattern across time that is symmetric between the two countries. When the infection peaks in country A, the planner responds by raising tariffs significantly above 0 in country A, while imposing a negative tariff in country B.

These tariffs are intriguing at a first pass because they encourage both countries to consume more of country A's goods, which transmits the pandemic via consumption- and labor-induced interactions in country A and via imports to country B. However, these health costs are dominated by the economic benefits — as the tariffs raise the terms of trade for country A during the peak of the infection, its households have higher income and enjoy a higher level of consumption. Similarly, when the second wave of infection hits country B, the planner reverses the tariffs in both countries, leading to more favorable terms of trade for country B and raising its households' consumption. The tariffs act as a lever to change the terms of trade. Note that the planner raises the terms of trade by more than 15 per cent in favor of country A during the peak of its pandemic, while they actually decrease under laisser-faire. This reversal of the terms of trade brought about by boosting tariffs allows risk-sharing between the two countries due to the asynchronous feature of the pandemic.

4.3 Government Policy in Nash Equilibrium

We next consider the case where each country's government determines its own domestic containment and tariff policies in order to maximize the welfare of their domestic households, defined as the weighted average of their lifetime utilities (29). We consider open-loop strategies, which is tantamount to assuming commitment and perfect foresight. This creates room for intertemporal tradeoffs of the "do-ut-des" sort: governments can agree in advance on future actions to smooth health shocks. However, this also creates the potential to create the "Prisonners'-Dilemma" type blockades of the traditional trade theory of tariffs.

Figure 5 reports the outcomes and Table 2 summarizes the basic statistics. To interpret the results, it is helpful to begin at the end. Once the pandemic is over (week 103 in country A, week 137 in country B), both governments impose a tariff of 23 per cent, due to the standard Nash logic that each country wants to boost its domestic employment and wages, given that the other country does so (second row, second panel). This logic interferes with the objective of smoothing intertemporal shocks during the pandemic. Still, the outcome is better than the laissez-faire outcome discussed above. In particular, Nash governments manage to flatten the curve - their infection peaks are lower and their pandemic durations are longer than with no policy at all. But while international cooperation lengthens the pandemic in country A by 9 weeks, Nash governments only achieve 6 weeks (see Table 2). Infection peaks and total death rates fare similarly.

The logic behind this coordination failure can again best be understood when looking at the first peak, in country A. During this wave, country B raises its tariff way above its standard trade-war levels, because this way it can boost its own production and at the same time keep infections from country out. As a reaction country A slashes its tariffs to levels even below 0, i.e., it provides import subsidies in order to compensate its reduced domestic consumption possibilities. Both actions in the end tilt the terms of trade against the infected country, thus amplifying the terms-of-trade problem under no policy by an order of magnitude (a deterioration by more than 30 per cent). Given this defensive policy by country A, country B's aggressive behavior is rational. On average, over the course of the pandemic, A's tariffs are below the stationary trade-war level, but their timing is wrong.

The timing problem is aggravated by a self-enforcing feedback mechanism between the policy variables at the disposal of each government. If a government imposes strict domestic containment measures (μ^k), this depresses all consumption in the country, including that of imports.[20] But with reduced imports the risk-sharing mechanism that would be needed to partially compensate for the local losses is weakened, which in turn necessitates even stronger domestic measures. This suggests that the interplay between the two policy variables makes matters worse in the case of uncoordinated governments.

4.4 Comparing Nash and Coordinated Policies

Figure 6 compares the equilibrium government policies and pandemic dynamics in the three cases discussed above. Both the Nash case and the Planner case feature similar paths of domestic containment policies, with high values during the peak of their infection waves and no action outside this period.

As argued earlier, the wave in country B is special as at that time country A has almost reached herd immunity. Under coordinated planning it is therefore possible to grant high import subsidies in country A (reducing tariffs below -40 per cent) to support country B's production and strengthen its terms of trade, while at the same time imposing drastic domestic containment measures in order to contain the pandemic in B. Hence, during the second peak, the planner imposes higher domestic containment measures in country B than the Nash government. During the first peak, which we argue should be viewed as the generic case in a world far from herd immunity, the reverse is true. As discussed above, the planner can get away with laxer domestic containment in A than under Nash because the dynamics of the pandemic make it possible to abandon the trade-war logic and modulate tariffs intertemporally, thus reducing the home bias and improving the terms of trade when A needs this most. As Figure 6 and Table 2 show, this positive spiral also reduces A's infections and ultimately its death toll.

These results highlight the contrast between health and economic externalities. Health externalities arise from the possibility that a country does too little to shut down its production and consumption activities, thus spreading the pandemic. Economic externalities arise from the possibility that a country will reduce its consumption of foreign goods in order to promote the interests of its own workers and firms. The coordinated planner fully internalizes this economic externality and uses tariffs to control the pandemic and smooth out its impact on both countries' economies. This way, international trade can lead to better risk-sharing and facilitates global health diversification. Importantly, the two externalities interact. When the disease hits one country the demand for its good collapses for health reasons, leading to a collapse of its price. This, however, triggers a demand effect in the less affected country, where

[20] Vivid realizations of this phenomenon are the images of endless queues of lorries at national borders or idle ports.

the risk of infection is overall lower, and thus provides a countervailing stimulus that is absent in the affected country. The government in the unaffected country reacts by increasing tariffs to contain that stimulus and at the same time benefit from domestic financial gain of tariffs. This leads to the apparently paradoxical situation, exhibited in the second row of Figure 6, that in Nash equilibrium imports in one country can peak when tariffs are the highest.

We disentangle these forces in the decomposition of the overall policy effect in Table 1. Panel (a) reports the welfare of the full benchmark case with pandemic and government policy. We decompose the households' utility loss in each country relative to the pre-pandemic level into two components: the welfare loss due to economic recession, and the welfare loss due to death. The former is the present value of the utility change in the consumption and labor of living households, from period 1 to the infinite future; the latter is the present value of the foregone utility due to death. Their sum is the total utility loss relative to the alternative world with no pandemic and no government tax and tariff.

We observe that the coordinated outcome alleviates both economic and death-related welfare losses relative to the Nash equilibrium. We normalize the utility numbers so that 0 represents the utility with no pandemic and no trade war. The planner lowers the utility loss due to death by partially shutting down the economy and causing a welfare loss due to economic shutdown, thereby achieving a trade-off between these two components by engaging in health and economic risk-sharing. Clearly, the social planner implements a different consumption-work-health tradeoff than that resulting from laissez-faire, with greater emphasis on health. In comparison, the Nash governments engage in the trade war, which leads to not only more utility loss due to death, but also more economic losses.

To put this result in a different perspective, Table 1 panel (b) reports the welfare calculation in a world with no pandemic, where the welfare loss from tariffs is 25.23 units. In the world with the pandemic, the welfare loss due to economic recession is even greater due to the governments' containment policies and households' precaution.

4.5 Containment Without Tariffs: The Case $\nu \equiv 0$

An interesting variant of our model obtains if we rule out tariffs, i.e., set $\nu \equiv 0$. This case certainly is realistic in some cases, as tariffs and other trade barriers are internationally regulated by trade agreements and cannot be changed flexibly in crises. Furthermore, in many parts of the world, most notably the European Union, tariffs have been abolished altogether.

We report the health and economic dynamics in this case in Figure 7, which mirrors Figure 6 of the full model and again compares laisser-faire, Nash equilibrium, and optimal coordination. Table 4 reports the corresponding statistics in greater detail. In this case, and different from the case $\nu > 0$, the domestic containment policies adopted under coordinated planning and in Nash equilibrium are qualitatively and quantitatively very similar, and so are the outcomes. In particular, governments in Nash equilibrium cannot use tariffs to counteract the risk-shifting policies that are optimal under coordination. Therefore, key variables such as the terms of trade now move very much alike under coordination and non-coordination. Thus, in terms of the observed dynamics, "Nash broadly gets it right". This, however, masks some interesting differences between the two settings. In particular, in country A the coordinated planner takes the externalities on future infections in country B into account and therefore imposes slightly stricter domestic containment policies than the Nash government. This leads to slightly fewer infections than under Nash. However, in country B the coordinated planner imposes less stringent domestic containment policies than the Nash government. This is because at that time the disease has largely run its course in country A and there are no more future externalities to be considered. The only relevant externality now is the static one resulting from the positive effect of domestic containment policies on infections in the other country through consumption. This positive externality is ignored by non-cooperative governments, which leads to too strict domestic measures. The Nash government in country B therefore gets the health-consumption tradeoff wrong in favor of too few infections. Even overall, as Table 4 shows, governments under non-coordination do better on the health front than under optimal coordination, because they put excessive weight on domestic containment. As discussed in the introduction, this is different when governments have both instruments, domestic containment and tariffs, at their disposal.

5 Generalized Lockdown, International Transmission Intensity, and Health Sector Overload

In this section, we discuss three important variations of our model that broaden the perspective on its interpretation. First, we present results for lockdowns which suppress productivity as well as consumption consumption; such lockdowns can be interpreted as "generalized lockdowns" wherein not just consumption but production is also restricted and impaired. Second, we discuss the contact intensity parameter π_4 that allows us to explore the implications of our model on merchandise vs services trade. Third, we discuss a variation in the model that can be interpreted as varying the congestion externality from a health-sector overload that allows us to understand the implications of the model for varying degrees of country development (on the health-care infrastructure front).

5.1 Generalized Lockdown through Productivity Suppression

At onset of the pandemic in March 2020, most countries adopted lockdowns that restricted both consumption and production barring the most essential services such as health and food delivery. Such generalized lockdowns appeared to have dramatic consequences on consumption and production of affected countries, even if there was global demand for some of the production in foreign countries. In contrast, by second waves (in 2020 or 2021), lockdowns were less generalized and more targeted; in particular, exports emerged as a potential way to keep domestic infected economy stronger by benefiting from demand in less- or non-infected parts of the world. Recognizing this, exports were included as "essential" services during the second waves.[21]

To study generalized lockdowns, we allow the domestic containment tax to also suppress productivity: $z_t^k = \bar{z}(1 - \mu_t^k)$. Figure 8 compares the coordinated outcome between our baseline model and the model with productivity suppression and Figure 9 compares the Nash outcome. When the government containment policy negatively affects productivity, the government is more reluctant to impose a strigent containment policy. As a result, the infection curve rises more drastically and the death toll is higher, both in the coordinated outcome and in the nash outcome. Furthermore, since using containment taxes is very costly, both the Nash and the planner outcomes use more aggressive tariff policies.

Table 5 compares the welfare differences of these cases. As with our benchmark model, the planner improves outcomes relative to Nash both on the economic and the health fronts. Table 6 reports statistics about the pandemic in the same format as the table for the benchmark specification. Relative to our benchmark model without production suppression, health outcomes are worse, with more deaths in both countries. This suggests that using more targeted lockdowns, rather than generalized lockdowns which suppress productivity and consumption, can lead to better global outcomes. Put another way, including export-oriented sectors as "essential" services that are allowed to operate during lockdowns could soften the economic blow of containment policies aimed at limiting spread of the pandemic via restrictions on domestic consumption.

5.2 Policy Variation with Varying Degree of Contact Intensity

While our modeling of trade so far carried the semantics of merchandise goods, in practice—depending on the country pairs—trade may also take the form of services. Especially in the

[21] A striking case of such classification was in India which faced a highly infection Delta variant of COVID-19 during April-June 2021, but yet on back of exports it managed to experience lockdown-related contraction that was far smaller than the one experienced during the generalized lockdown of March-May 2020 (when it in fact faced a lower infection rate).

context of the pandemic, services trade relating to tourism, travel, transport, etc., carries a particular importance as it has much greater contact intensity than trade in other services (such as technology services) and merchandise goods. Consistent with this view, travel and transport have been the most adversely affected services sector during the pandemic, and overall services trade in March 2021 remained below its pre-pandemic levels unlike merchandise trade that had recovered more fully (World Bank, "Trade Watch", June 2021). This distinction between merchandise and services trade is also germane from the standpoint of governments having adjusted "tariffs" during the pandemic.

Within merchandise trade, an important trade restriction has been in the form of port shutdowns. Infections at docks are a relevant concern given the high contact intensity involved in the jobs there. As a result of countries adopting port shutdowns, notably in China but also elsewhere, container throughput capacity collapsed to 70% during the pandemic, with benchmark costs of shipping a container between China and the United States tripling during the pandemic in some cases[22] and rising to 10 times higher than the pre-pandemic level in other cases. Nevertheless, many contend that direct revisions to tariffs due to government interventions have been somewhat limited in the context of merchandise goods. This is, however, certainly not the case in services trade, notably in travel. Border controls and restrictions to tourism have been ramped up dramatically, some might contend even in a draconian manner when they have taken the form of border shutdowns, even within otherwise integrated regions such as states in the case of United States and member countries in the case of the Eurozone. While modeling the full richness of merchandise versus services trade is beyond the scope of this paper, we derive some conclusions to our results implied by their difference by considering a comparative static in π_4, which measures the intensity in the transmission of disease between households in different countries due to the consumption of foreign goods. In particular, varying π_4 to higher levels can be considered as shifting focus towards countries engaged in more contact-intensive trade such as services versus merchandise, or within services, tourism versus technology services.

In Figures 10 and 11, we vary the parameter value of π_4 and report the equilibrium outcomes of coordinated and non-cooperative governments, respectively. We consider the benchmark case, the case in which the π_4 is five-fold higher, and the case in which the π_4 is ten-fold higher. A higher π_4 means that the pandemic is transmitted faster across countries for a given level of international trade. Roughly speaking, international transmission in the benchmark case is about 1/100 as strong as domestic transmission. So, when we raise the π_4 by ten-fold, the international transmission is about 1/10 as strong as the domestic transmission.

[22] See, for example, China's Port Shutdown Raises Fears of Closures Worldwide, Bloomberg Economics, August 12, 2021: https://www.bloomberg.com/news/articles/2021-08-12/massive-china-port-shutdown-raises-fears-of-closures-worldwide

In both the coordinated planning equilibrium and the Nash equilibrium, the faster transmission means that the infection peaks in the two countries are temporally closer. If we raise the international transmission coefficient ten fold, the two infection peaks in the absence of government policies are 12 weeks apart instead of 27 weeks apart. Table 8 reports these statistics in the same format as the table for the benchmark specification.

We further note that the government policies remain qualitatively similar to those in the benchmark case. In the coordinated planning equilibrium, each government imposes domestic containment taxes during the peak of its local infection. It also raises its import tariffs during the peak of its local infection, and lowers its import tariffs during the peak of the other country's local infection. As we discussed above, these coordinated tariff policies allow the country that is experiencing the pandemic at the moment to benefit from a stronger term of trade and therefore a higher welfare, acting as an risk-sharing scheme.

In the Nash equilibrium, each government imposes domestic containment taxes during the peak of its local infection. Its import tariff is unconditionally high, except during the peak of its local infection. This pattern is also qualitatively consistent with our finding in the benchmark specification.

Table 7 compares the welfare differences of these cases. We are interested in whether a stronger international transmission limits the scope of international coordination by governments. To measure this scope, we compare the difference between the welfare obtained under coordinated and Nash policies. We find that the welfare gain due to less death under coordinated policies relative to Nash policies is 1.21 for country A and 1.87 for country B in the baseline case. When we raise the international transmission coefficient by ten-fold, this welfare gain goes up to 1.79 for country A and goes down to 0.63 for country B. To understand this difference, we note that, given the pandemic starts in country A, a stronger international transmission is always a bad news for country B, as it allows less time for country B to flatten the curve. As a result, country B suffers higher death and there is less its government can do even under coordinated policies. On the other hand, since the scope of protecting country B is more limited, country A has to make less sacrifice, which results in a relative welfare gain under coordinated policies when the transmission coefficient is higher.

5.3 Policy Variation with Varying Degree of Healthcare Congestion

Another interesting application of our international model of the pandemic is to consider trade between advanced economies (AEs) versus emerging markets (EMs). While at the onset of the pandemic, EMs were thought to be less exposed due to younger and less obese populations, in many cases the fallout of the pandemic has been worse in the EMs, notably due to their limited capacity in health infrastructure. For instance, in terms of hospital beds per 1,000 people, World Bank's most recently available statistics show that India has 0.5, Philippines 1, United States 2.9, China 4.3 and Japan 13.4, the statistics being even more disperse in case of Intensive Care Unit (ICU) beds.

The lack of limited healthcare capacity has been found during the pandemic to extend beyond just hospital and ICU beds to availability of medical equipment and oxygen supply, implying that the realized infection fatality rate in EMs can be country-specific due to "congestion externalities" from healthcare overload rather than being just disease-specific. While there are several other differences in EMs relative to AEs, due to the former's greater density of population, higher contact-intensity nature of low-paying jobs, and higher imports component in the consumption basket, we focus on varying the extent of congestion externality to derive model's implications.[23]

In Figures 12 and 13, we either raise the congestion parameter from the benchmark of 0.05 to 0.075, or lower the congestion parameter from the benchmark of 0.05 to 0.025. Naturally, we find that a higher congestion parameter leads to more death, and governments impose more stringent domestic containment tax as a response in both the coordinated and the Nash equilibria. On the other hand, the governments' tariff policies remain similar. Interestingly, we also note that as more people die under a higher congestion parameter and the infection rate goes down. Table 10 reports these statistics in the same format as the table for the benchmark specification.

Table 9 compares the welfare differences of these cases. A higher congestion parameter makes it more important to contain the pandemic, therefore increasing the scope of government policies. As a result, the welfare gain due to less death in the coordinated policies relative to the Nash policies is increasing as the congestion parameter is higher.

6 Conclusion

In this paper, we have developed a model of epidemiology and international trade to study how international coordination and the lack thereof influences the impact of government policies on health and economic outcomes. A major insight from our work concerns the interplay between domestic health policies and international trade policies allows to dynamically modulate the terms of trade and thus shift consumption and infection patterns internationally as a function of the global state of the pandemic. When policies are modulated, dynamics are efficient generating a variation in terms of trade that favor the country experiencing peaks of their

[23] Our model is rendered inordinately more complex than the present formulation if we allowed for country heterogeneity among the two countries; hence, we limit our present exercise to simply varying the congestion externality for both countries.

infections waves; uncoordinated policies aggravate overall outcomes by achieving exactly the opposite.

In ongoing work, we are generalizing the model to study the role of non-tariff barriers during a pandemic in affecting international supply chains, which seem disrupted far more than originally evisaged at the time of outbreak of the COVID-19 pandemic. An important generalization is that traded goods can be intermediary inputs into domestic production functions that produce the final good consumed in each country. We hope that our analysis will ultimately be able to shed light on the important general question of the costs and benefits of coordination of local health and economic policies in the context of possible supply-chain disruptions, be it between different sovereign governments, between states in a federation, or within the European Union.

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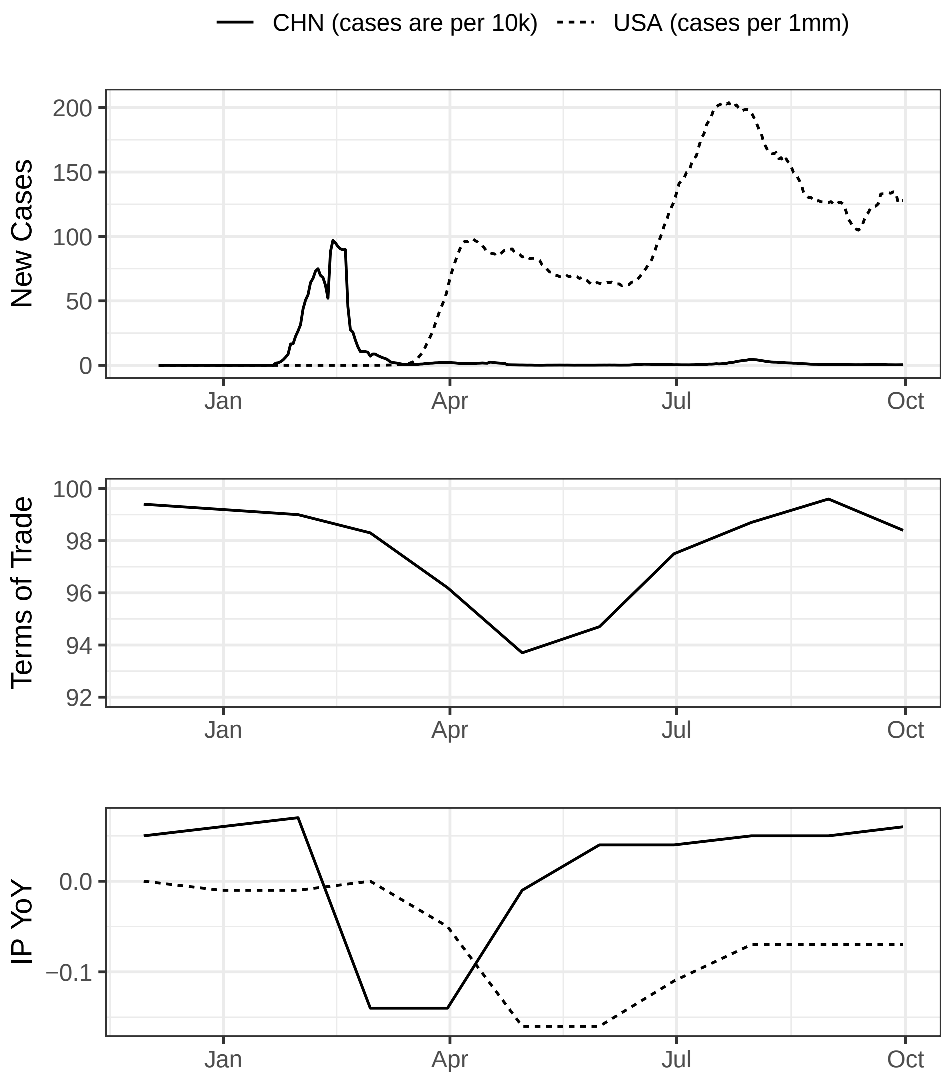
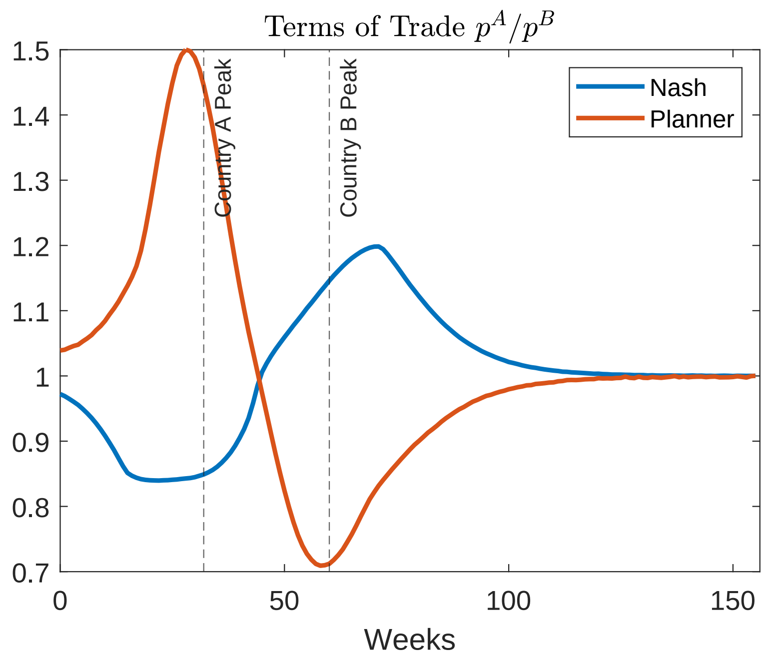
Figure 1: Pandemic and Economic Outcomes in China and the U.S.



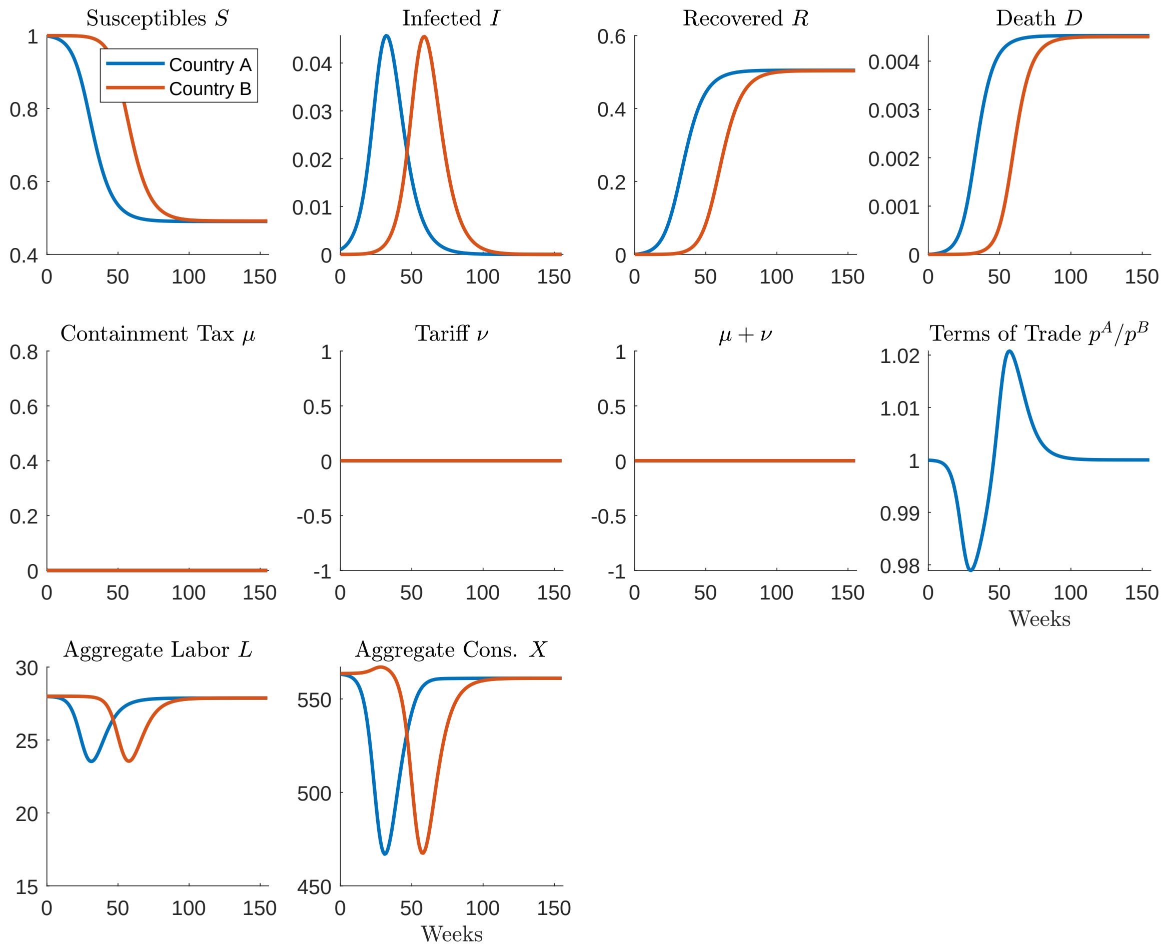
Note: Health and economic outcomes in China and the United States during the 2020 pandemic. Daily new cases for China are per 10,000 people and per 1,000,000 for the United States. Terms of trade is the price index of US exports to China divided by the price index of imports by the US from China. Industrial production is measured year-over-year.

Figure 2: Terms of Trade With Coordinated and Uncoordinated Policy



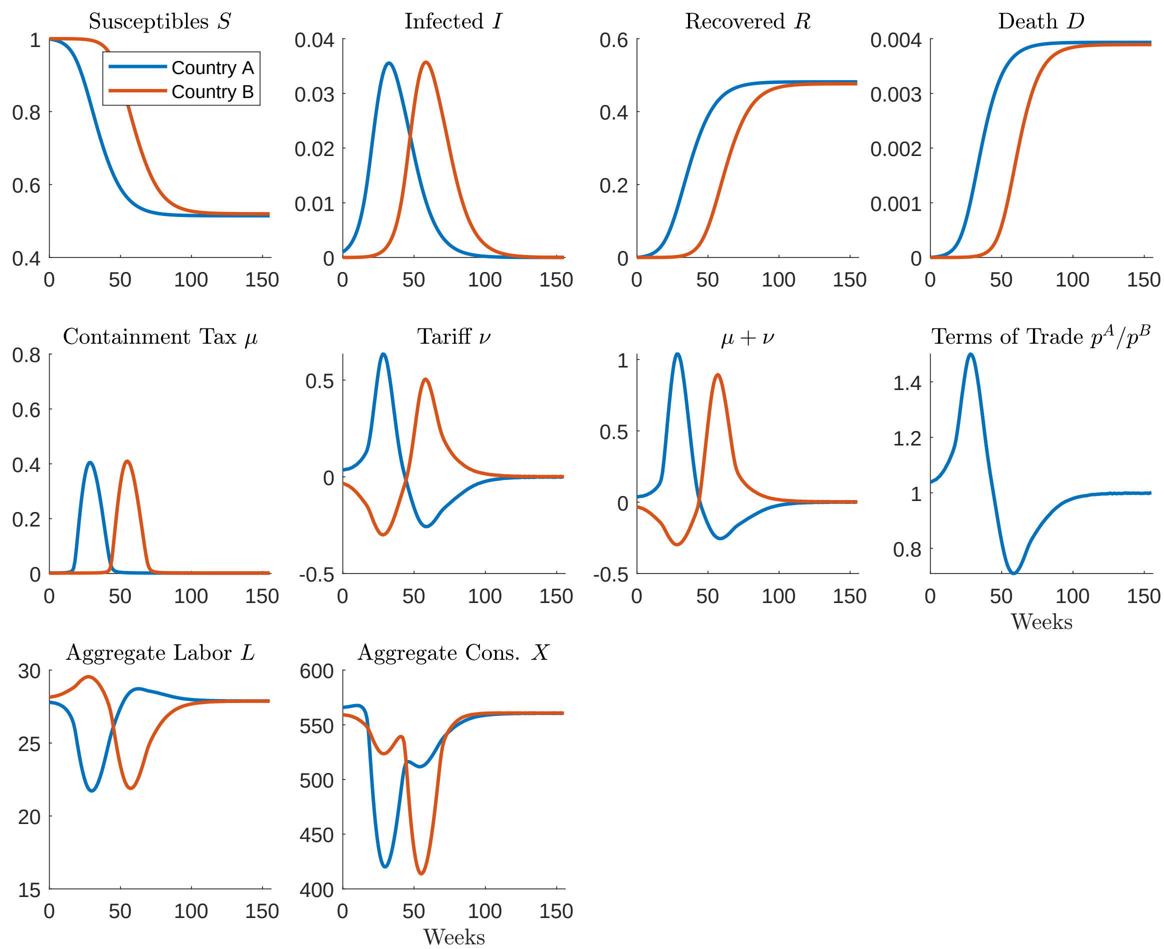
Note: Terms of Trade in uncoordinated (Nash) and coordinated (Planner) equilibrium. The dashed lines specify the approximate peak of maximum infections in country A and country B.

Figure 3: Benchmark SIR Dynamics



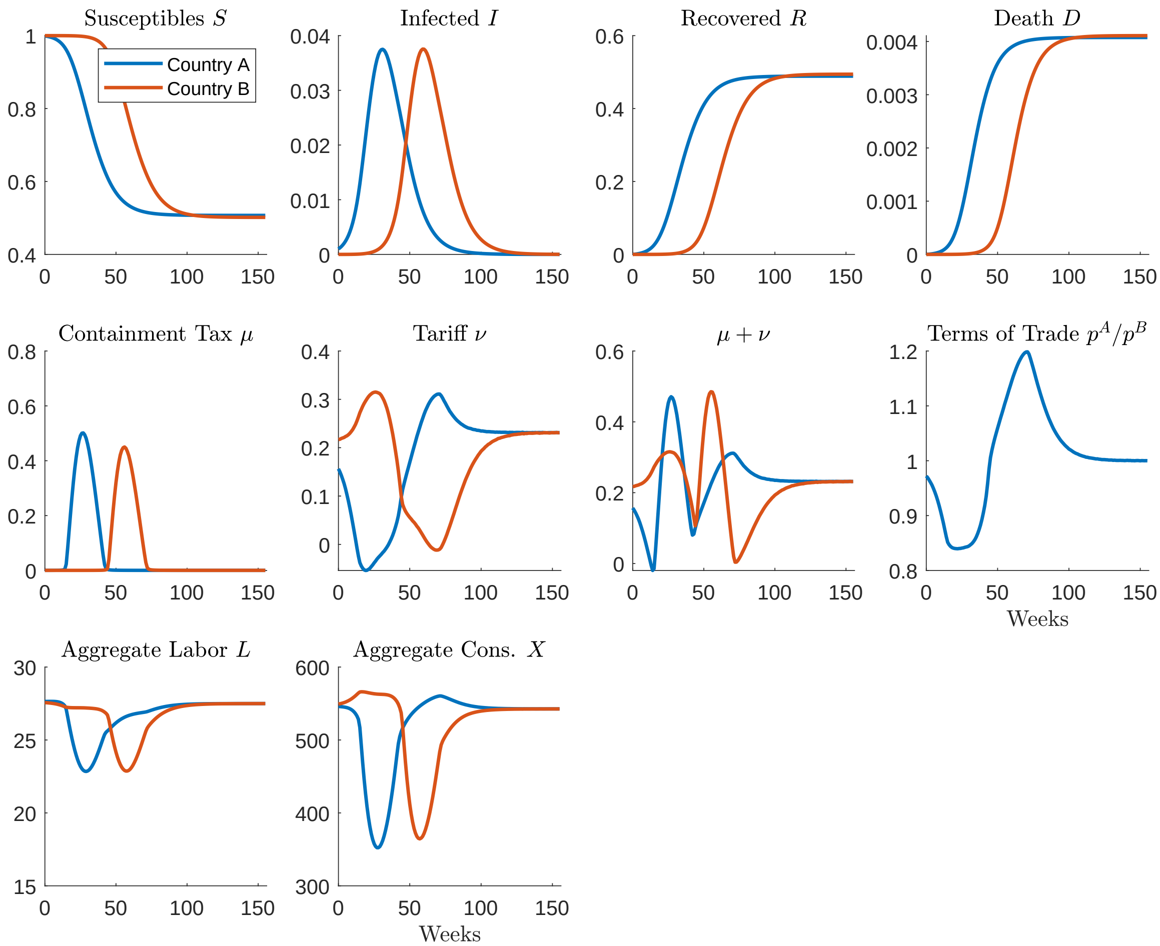
Note: Benchmark model with international transmission of pandemic. No government domestic containment policies or tariffs.

Figure 4: Coordinated Planning Equilibrium Outcomes



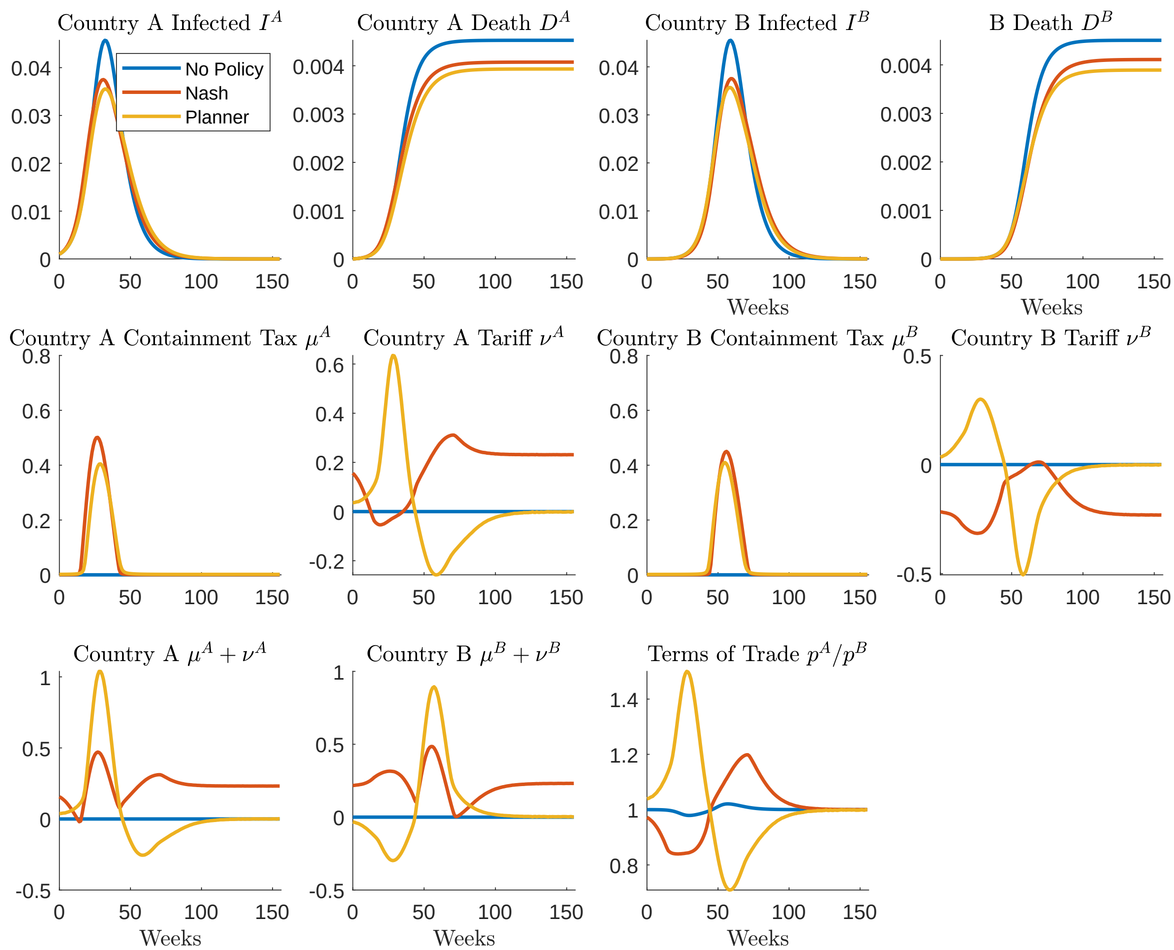
Note: Benchmark model with international transmission of pandemic. Equilbirium domestic containment policies and tariffs are determined by a global social planner that maximizes the sum of both countries' welfare.

Figure 5: Nash Equilibrium Outcomes



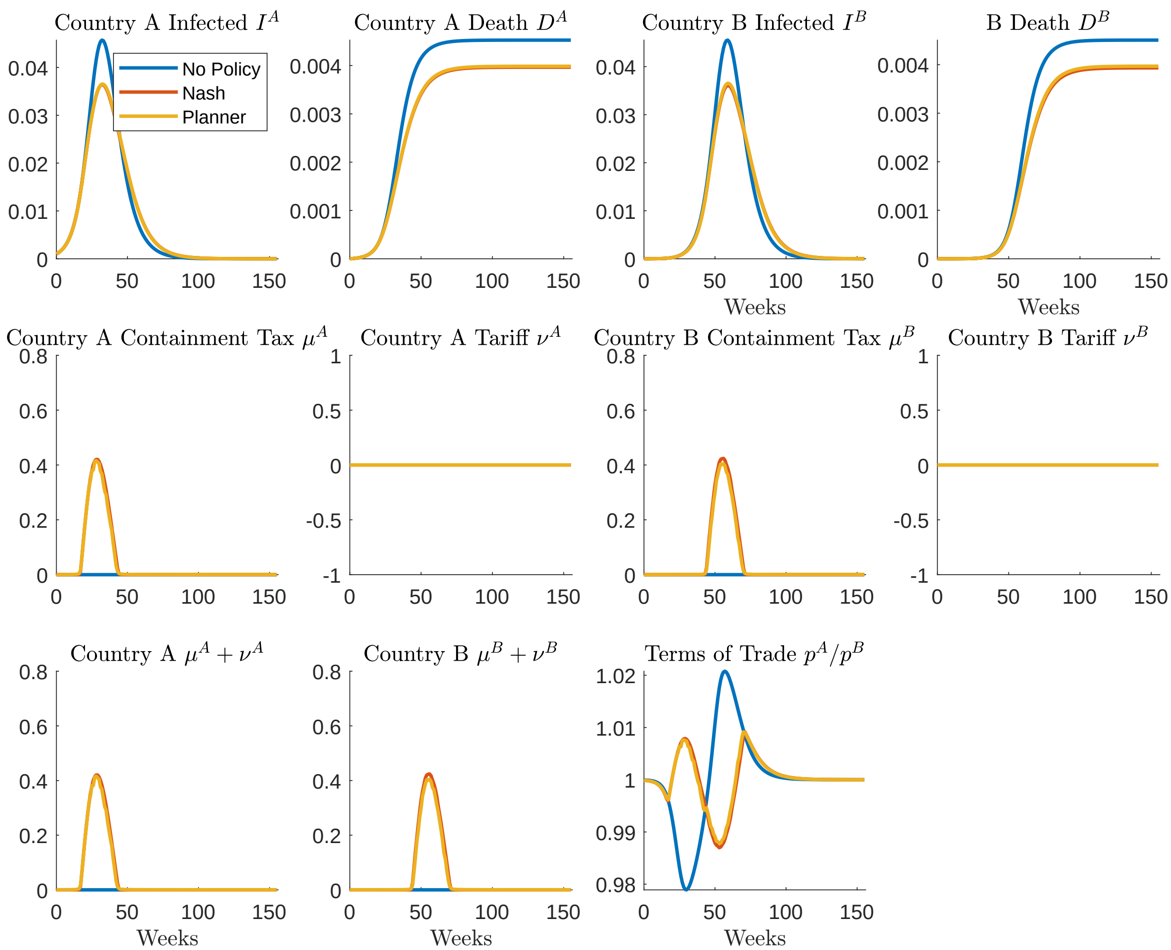
Note: Benchmark model with international transmission of pandemic. Equilibrium domestic containment policies and tariffs are the outcome of a Nash game between the two countries.

Figure 6: Comparing Equilibrium Policies and Outcomes



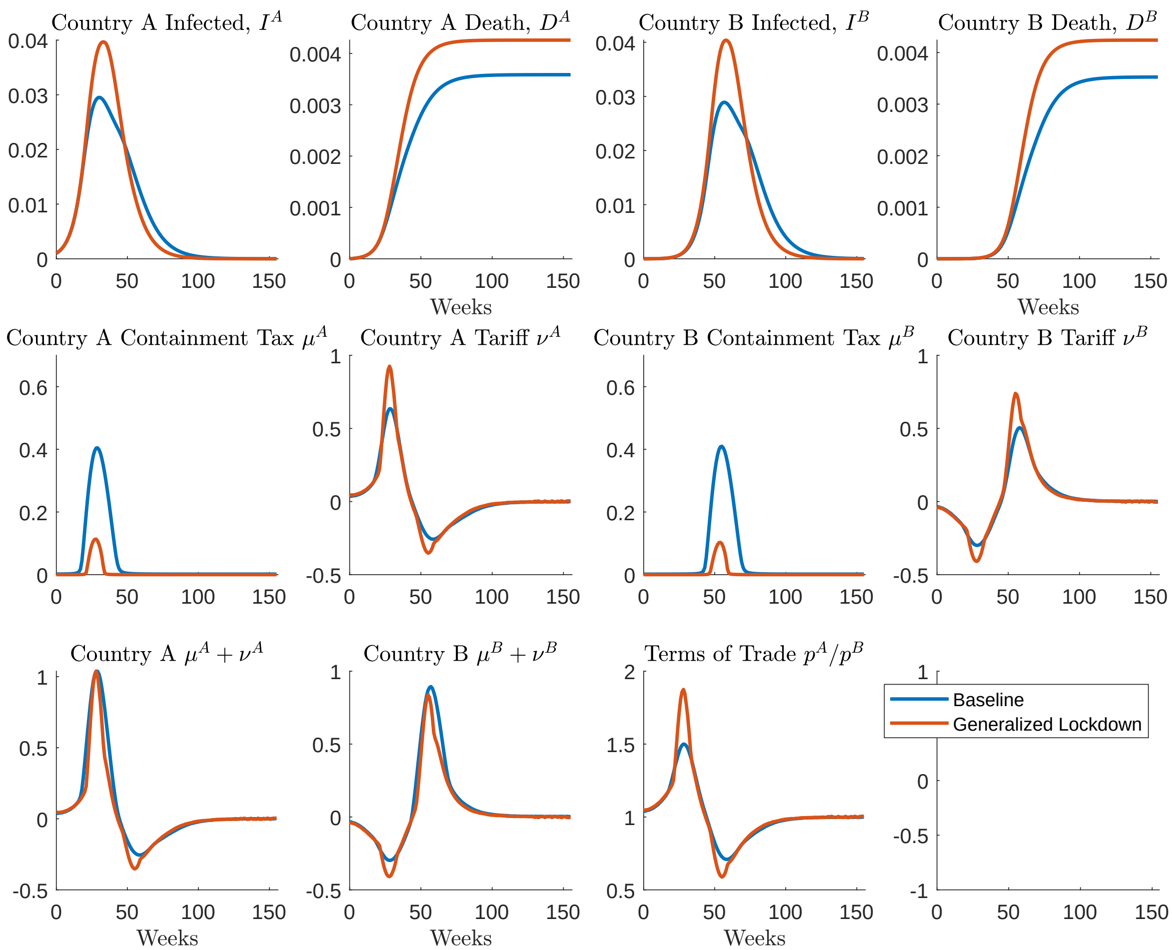
Note: Comparison of SIR dynamics, government policies, and economic outcomes in three cases: benchmark, Nash, and Planner. In the no policy case there are no domestic containment policies. In the Nash case, equilibrium domestic containment policies and tariffs are the outcome of a Nash game between the two countries. In the planner case, equilbrium domestic containment policies and tariffs are determined by a global social planner that maximizes the sum of both countries welfare.

Figure 7: Comparing Equilibrium Policies and Outcomes, No Tariff



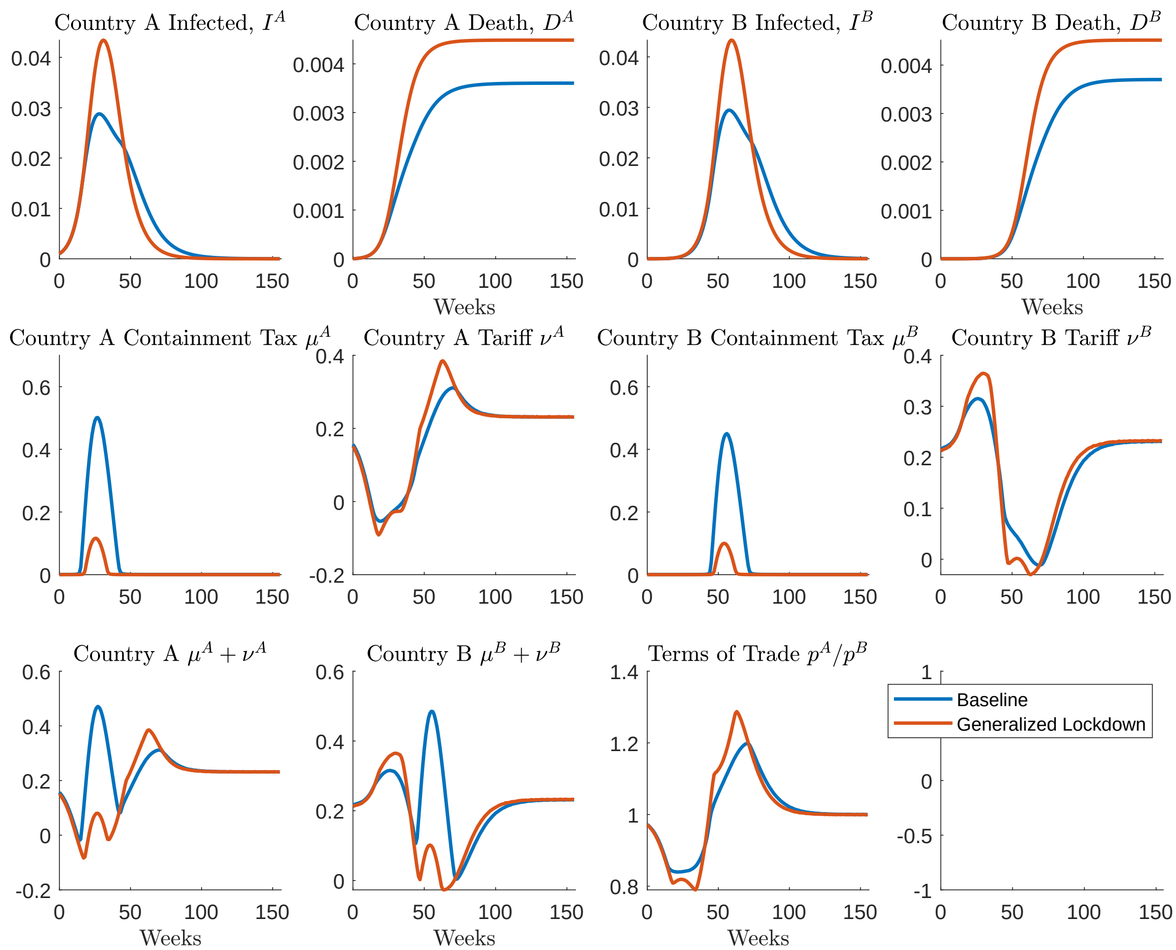
Note: Comparison of SIR dynamics, government policies, and economic outcomes in three cases: benchmark, Nash, and Planner. We study the case in which no tariff can be imposed.

Figure 8: Production Supression, Coordinated Planning Equilibrium



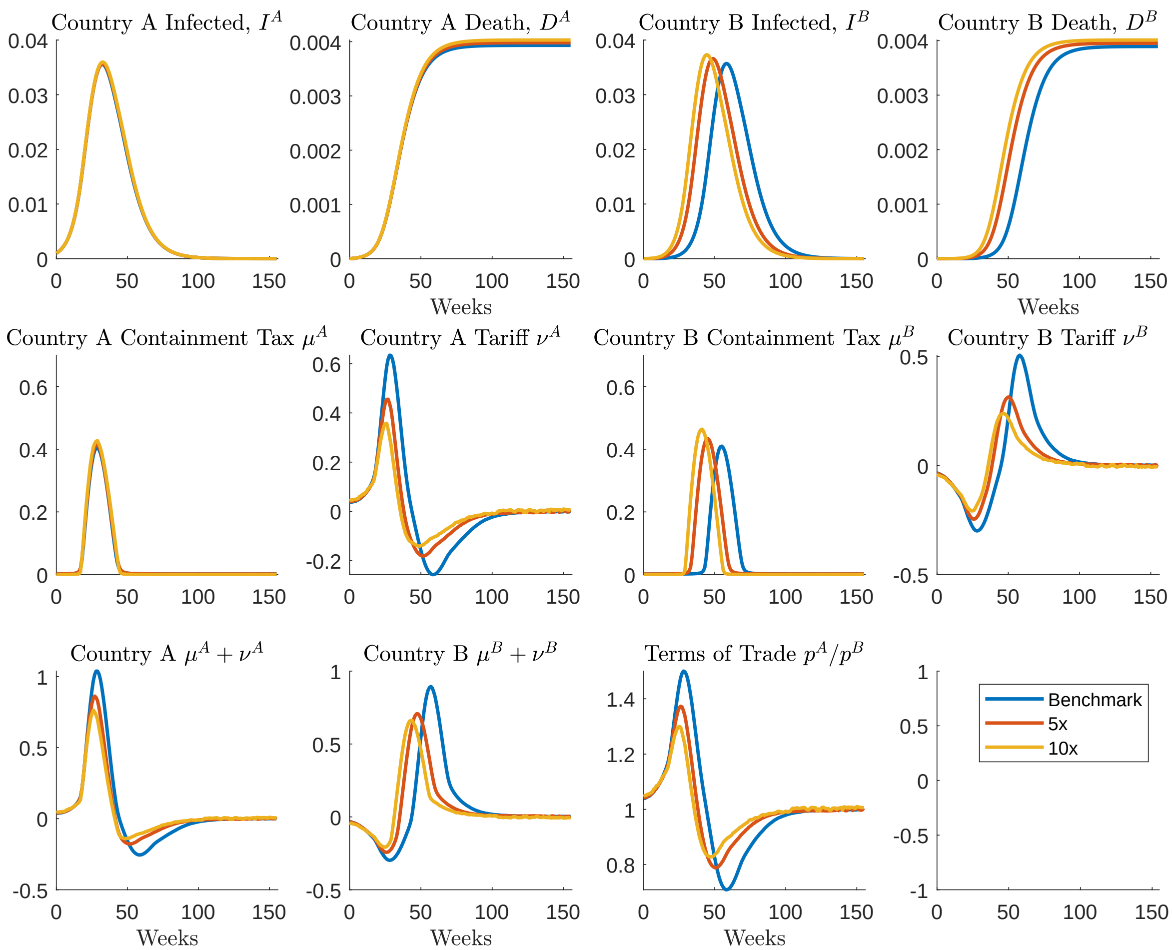
Note: Comparison of SIR dynamics, government policies, and economic outcomes in coordinated planning equilibria. In this variation, we allow the government containment policy to affect local productivity level.

Figure 9: Production Supression, Nash Equilibrium



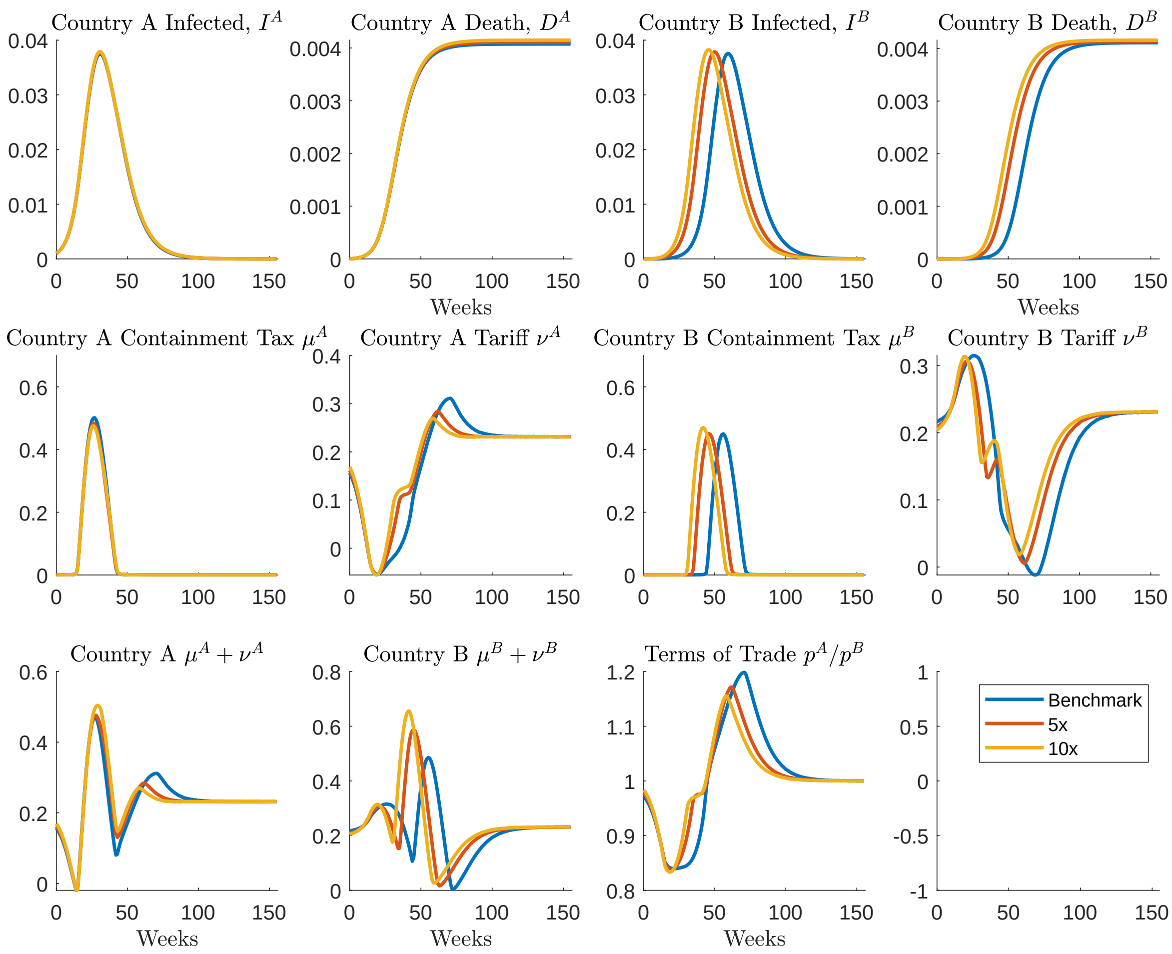
Note: Comparison of SIR dynamics, government policies, and economic outcomes in Nash equilibria. In this variation, we allow the government containment policy to affect local productivity level.

Figure 10: Varying International Transmission π_4, Coordinated Planning Equilibrium



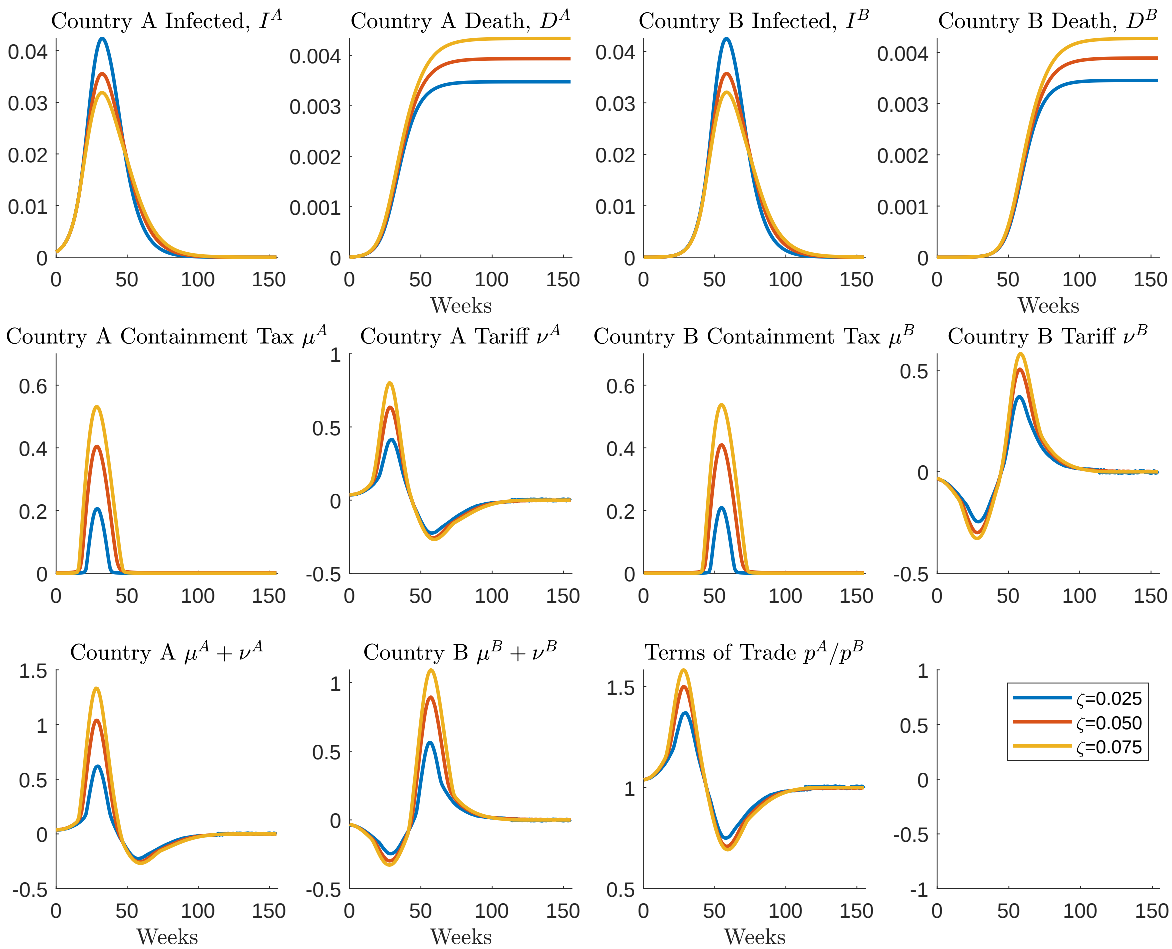
Note: Comparison of SIR dynamics, government policies, and economic outcomes in coordinated planning equilibria. We vary the parameter π_4 that governs the intensity of international transmission of disease.

Figure 11: Varying International Transmission π_4, Nash Equilibrium



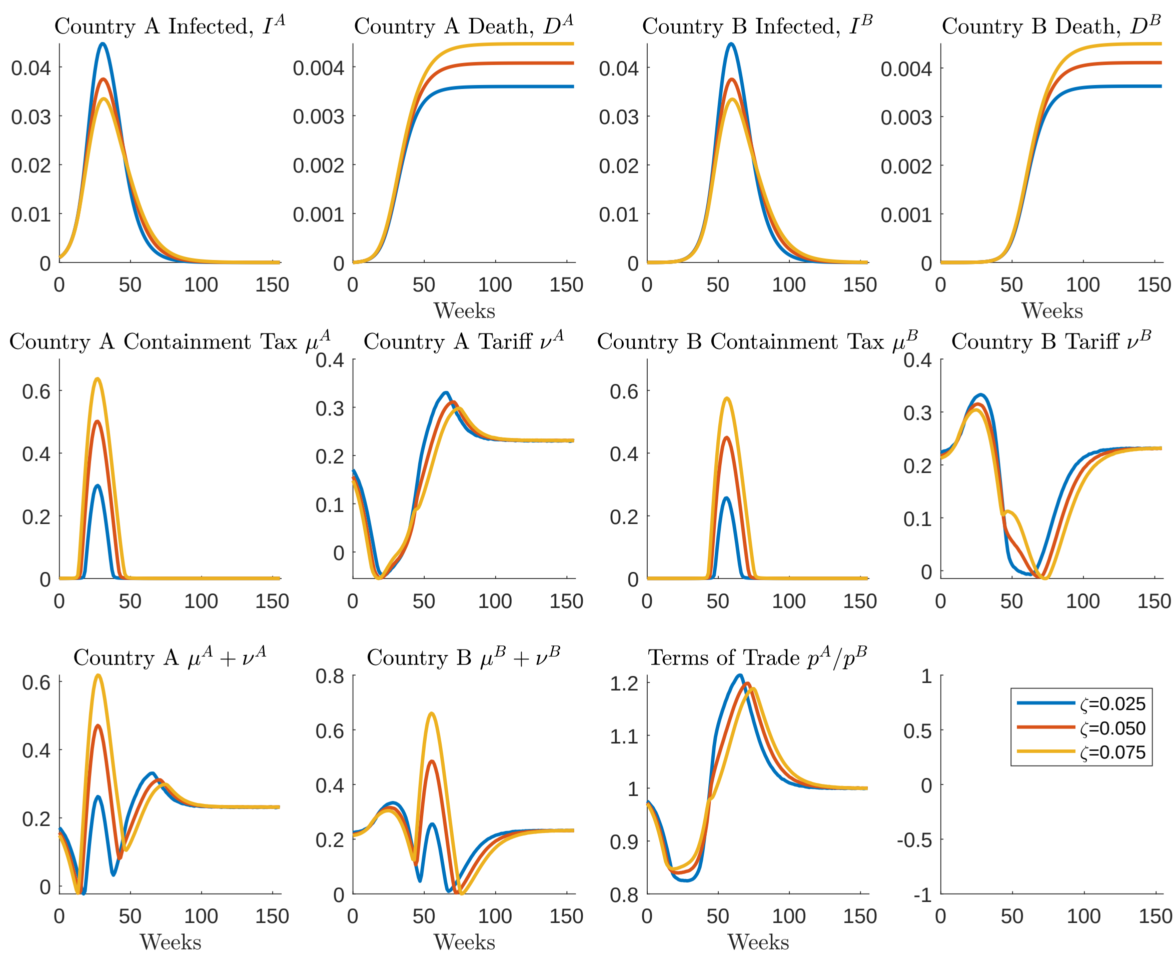
Note: Comparison of SIR dynamics, government policies, and economic outcomes in Nash equilibria. We vary the parameter π_4 that governs the intensity of international transmission of disease.

Figure 12: Varying Congestion Rate ζ, Coordinated Planning Equilibrium



Note: Comparison of SIR dynamics, government policies, and economic outcomes in coordinated planning equilibria. We vary the parameter ζ that governs the increase in death rate due to congestion in hospital. The benchmark is $\zeta = 0.050$.

Figure 13: Varying Congestion Rate ζ, Nash Equilibrium



Note: Comparison of SIR dynamics, government policies, and economic outcomes in Nash equilibria. We vary the parameter ζ that governs the increase in death rate due to congestion in hospital. The benchmark is $\zeta = 0.050$.

Table 1: Welfare Decomposition

We report the welfare loss relative to the steady-state level without pandemic and policy. We decompose the welfare loss in each country into two components. The *economic* loss is the present value of the utility loss of living households due to changes in consumption and labor during the pandemic episode, and the *death* loss is the present value of the foregone utility due to death.

Panel (a): With Pandemic						
	Country A			Country B		
	Total	Economic	Death	Total	Economic	Death
No Policy	-33.62	-0.88	-32.74	-32.79	-0.87	-31.92
Nash	-58.35	-28.97	-29.37	-57.34	-28.38	-28.96
Planner	-32.61	-4.21	-28.40	-31.52	-3.98	-27.54
Planner - Nash	25.74	24.76	0.98	25.82	24.40	1.42
Panel (b): No Pandemic						
	Country A			Country B		
	Total	Economic	Death	Total	Economic	Death
No Policy	0.00	0.00	0.00	0.00	0.00	0.00
Nash	-25.23	-25.23	0.00	-25.23	-25.23	0.00
Planner	0.00	0.00	0.00	0.00	0.00	0.00
Planner - Nash	25.23	25.23	0.00	25.23	25.23	0.00

Table 2: Health Dynamics

We report some statistics about infection and death rates in both countries.

Benchmark Case			
	No Policy	Nash	Planner
Week of infection peak A	33.00	32.00	33.00
Week of infection peak B	60.00	60.00	59.00
Level of peak infection A (per 1000 households)	45.72	37.51	35.55
Level of peak infection B (per 1000 households)	45.52	37.54	35.71
Last week of pandemic A (over 0.01% infected)	97.00	106.00	107.00
Last week of pandemic B (over 0.01% infected)	122.00	135.00	132.00
Overall deaths A (per 1000 households)	4.53	4.08	3.93
Overall deaths B (per 1000 households)	4.51	4.11	3.89

Table 3: Welfare Decomposition, No Tariff

We report the welfare loss relative to the steady-state level without pandemic and policy. We decompose the welfare loss in each country into two components. The *economic* loss is the present value of the utility loss of living households due to changes in consumption and labor during the pandemic episode, and the *death* loss is the present value of the foregone utility due to death.

No Tariff						
	Country A			Country B		
	Total	Economic	Death	Total	Economic	Death
No Policy	-33.62	-0.88	-32.74	-32.79	-0.87	-31.92
Nash	-32.71	-4.06	-28.65	-31.84	-4.02	-27.82
Planner	-32.72	-3.93	-28.79	-31.86	-3.79	-28.07
Planner - Nash	-0.01	0.12	-0.14	-0.02	0.23	-0.25

Table 4: Health Dynamics, No Tariff

We report some statistics about infection and death rates in both countries.

No Tariff			
	No Policy	Nash	Planner
Week of infection peak A	33.00	33.00	33.00
Week of infection peak B	60.00	60.00	60.00
Level of peak infection A (per 1000 households)	45.72	36.41	36.53
Level of peak infection B (per 1000 households)	45.52	36.10	36.54
Last week of pandemic A (over 0.01% infected)	97.00	106.00	106.00
Last week of pandemic B (over 0.01% infected)	122.00	132.00	131.00
Overall deaths A (per 1000 households)	4.53	3.97	3.99
Overall deaths B (per 1000 households)	4.51	3.93	3.97

Table 5: Welfare Decomposition, Generalized Lockdown

We report the welfare loss relative to the steady-state level without pandemic and policy. We decompose the welfare loss in each country into two components. The *economic* loss is the present value of the utility loss of living households due to changes in consumption and labor during the pandemic episode, and the *death* loss is the present value of the foregone utility due to death.

Generalized Lockdown						
	Country A			Country B		
	Total	Economic	Death	Total	Economic	Death
No Policy	-33.62	-0.88	-32.74	-32.79	-0.87	-31.92
Nash	-59.48	-27.11	-32.37	-58.48	-26.62	-31.86
Planner	-33.25	-2.47	-30.77	-32.11	-2.06	-30.05
Planner - Nash	26.23	24.63	1.60	26.37	24.56	1.82

Table 6: Health Dynamics, Generalized Lockdown

We report some statistics about infection and death rates in both countries.

Generalized Lockdown			
	No Policy	Nash	Planner
Week of infection peak A	33.00	32.00	34.00
Week of infection peak B	60.00	60.00	59.00
Level of peak infection A (per 1000 households)	45.72	43.42	39.68
Level of peak infection B (per 1000 households)	45.52	43.31	40.40
Last week of pandemic A (over 0.01% infected)	97.00	98.00	101.00
Last week of pandemic B (over 0.01% infected)	122.00	127.00	125.00
Overall deaths A (per 1000 households)	4.53	4.49	4.26
Overall deaths B (per 1000 households)	4.51	4.52	4.24

Table 7: Welfare Decomposition, Varying International Transmission π_4

We report the welfare loss relative to the steady-state level without pandemic and policy. We decompose the welfare loss in each country into two components. The *economic* loss is the present value of the utility loss of living households due to changes in consumption and labor during the pandemic episode, and the *death* loss is the present value of the foregone utility due to death.

Panel (a): 1 times π_4, Baseline						
	Country A			Country B		
	Total	Economic	Death	Total	Economic	Death
No Policy	-33.62	-0.88	-32.74	-32.79	-0.87	-31.92
Nash	-58.35	-28.97	-29.37	-57.34	-28.38	-28.96
Planner	-32.61	-4.21	-28.40	-31.52	-3.98	-27.54
Planner - Nash	25.74	24.76	0.98	25.82	24.40	1.42
Panel (b): 5 times π_4						
	Country A			Country B		
	Total	Economic	Death	Total	Economic	Death
No Policy	-33.93	-0.88	-33.05	-33.71	-0.91	-32.80
Nash	-58.56	-28.93	-29.63	-57.82	-28.49	-29.32
Planner	-33.01	-4.28	-28.73	-32.42	-4.24	-28.18
Planner - Nash	25.55	24.66	0.90	25.40	24.25	1.15
Panel (c): 10 times π_4						
	Country A			Country B		
	Total	Economic	Death	Total	Economic	Death
No Policy	-34.38	-0.89	-33.49	-34.58	-0.96	-33.62
Nash	-58.97	-29.07	-29.90	-58.32	-28.71	-29.61
Planner	-33.22	-4.13	-29.10	-33.19	-4.50	-28.68
Planner - Nash	25.75	24.95	0.80	25.14	24.21	0.93

Table 8: Health Dynamics, Varying International Transmission π_4

We report some statistics about infection and death rates in both countries.

Panel (a): 1 times π_4, Baseline			
	No Policy	Nash	Planner
Week of infection peak A	33.00	32.00	33.00
Week of infection peak B	60.00	60.00	59.00
Level of peak infection A (per 1000 households)	45.72	37.51	35.55
Level of peak infection B (per 1000 households)	45.52	37.54	35.71
Last week of pandemic A (over 0.01% infected)	97.00	106.00	107.00
Last week of pandemic B (over 0.01% infected)	122.00	135.00	132.00
Overall deaths A (per 1000 households)	4.53	4.08	3.93
Overall deaths B (per 1000 households)	4.51	4.11	3.89
Panel (b): 5 times π_4			
	No Policy	Nash	Planner
Week of infection peak A	33.00	32.00	34.00
Week of infection peak B	50.00	51.00	50.00
Level of peak infection A (per 1000 households)	45.86	37.71	35.63
Level of peak infection B (per 1000 households)	46.83	37.87	36.60
Last week of pandemic A (over 0.01% infected)	97.00	108.00	108.00
Last week of pandemic B (over 0.01% infected)	111.00	125.00	122.00
Overall deaths A (per 1000 households)	4.57	4.11	3.98
Overall deaths B (per 1000 households)	4.59	4.13	3.95
Panel (c): 10 times π_4			
	No Policy	Nash	Planner
Week of infection peak A	33.00	32.00	34.00
Week of infection peak B	45.00	47.00	45.00
Level of peak infection A (per 1000 households)	46.20	37.90	35.96
Level of peak infection B (per 1000 households)	48.32	38.25	37.30
Last week of pandemic A (over 0.01% infected)	97.00	108.00	108.00
Last week of pandemic B (over 0.01% infected)	106.00	121.00	117.00
Overall deaths A (per 1000 households)	4.63	4.15	4.03
Overall deaths B (per 1000 households)	4.69	4.16	4.01

Table 9: Welfare Decomposition, Varying Congestion Intensity ζ

We report the welfare loss relative to the steady-state level without pandemic and policy. We decompose the welfare loss in each country into two components. The *economic* loss is the present value of the utility loss of living households due to changes in consumption and labor during the pandemic episode, and the *death* loss is the present value of the foregone utility due to death.

Panel (a): $\zeta = 0.025$, *Low Congestion*						
	Country A			Country B		
	Total	Economic	Death	Total	Economic	Death
No Policy	-27.29	-0.67	-26.62	-26.64	-0.66	-25.98
Nash	-52.79	-26.84	-25.95	-51.97	-26.39	-25.58
Planner	-27.05	-1.93	-25.12	-26.22	-1.75	-24.46
Planner - Nash	25.74	24.91	0.83	25.75	24.64	1.11
Panel (b): $\zeta = 0.050$, *Baseline*						
	Country A			Country B		
	Total	Economic	Death	Total	Economic	Death
	total	economy	death	total	economy	death
No Policy	-33.62	-0.88	-32.74	-32.79	-0.87	-31.92
Nash	-58.35	-28.97	-29.37	-57.34	-28.38	-28.96
Planner	-32.61	-4.21	-28.40	-31.52	-3.98	-27.54
Planner - Nash	25.74	24.76	0.98	25.82	24.40	1.42
Panel (c): $\zeta = 0.075$, *High Congestion*						
	Country A			Country B		
	Total	Economic	Death	Total	Economic	Death
No Policy	-39.17	-1.09	-38.08	-38.18	-1.07	-37.10
Nash	-62.94	-30.75	-32.20	-61.74	-30.07	-31.68
Planner	-36.94	-5.67	-31.27	-35.84	-5.62	-30.23
Planner - Nash	26.00	25.07	0.93	25.90	24.45	1.45

Table 10: Health Dynamics, Varying Congestion Intensity ζ

We report some statistics about infection and death rates in both countries.

Panel (a): $\zeta = 0.025$, Low Congestion			
	No Policy	Nash	Planner
Week of infection peak A	34.00	31.00	33.00
Week of infection peak B	60.00	60.00	59.00
Level of peak infection A (per 1000 households)	48.50	44.74	42.37
Level of peak infection B (per 1000 households)	48.34	44.84	42.52
Last week of pandemic A (over 0.01% infected)	94.00	97.00	99.00
Last week of pandemic B (over 0.01% infected)	119.00	125.00	123.00
Overall deaths A (per 1000 households)	3.68	3.60	3.48
Overall deaths B (per 1000 households)	3.67	3.63	3.45
Panel (b): $\zeta = 0.050$, Baseline			
	No Policy	Nash	Planner
Week of infection peak A	33.00	32.00	33.00
Week of infection peak B	60.00	60.00	59.00
Level of peak infection A (per 1000 households)	45.72	37.51	35.55
Level of peak infection B (per 1000 households)	45.52	37.54	35.71
Last week of pandemic A (over 0.01% infected)	97.00	106.00	107.00
Last week of pandemic B (over 0.01% infected)	122.00	135.00	132.00
Overall deaths A (per 1000 households)	4.53	4.08	3.93
Overall deaths B (per 1000 households)	4.51	4.11	3.89
Panel (c): $\zeta = 0.075$, High Congestion			
	No Policy	Nash	Planner
Week of infection peak A	33.00	32.00	33.00
Week of infection peak B	60.00	61.00	59.00
Level of peak infection A (per 1000 households)	43.46	33.49	31.89
Level of peak infection B (per 1000 households)	43.23	33.47	32.07
Last week of pandemic A (over 0.01% infected)	99.00	114.00	114.00
Last week of pandemic B (over 0.01% infected)	124.00	143.00	139.00
Overall deaths A (per 1000 households)	5.27	4.47	4.33
Overall deaths B (per 1000 households)	5.24	4.50	4.28

A Model Appendix

A.1 The Static Model

Without pandemics, the model boils down to an essentially static two-country macro model. This is because, in order to focus on the epidemiological dynamics, in (25) we have ruled out economic dynamics. As a benchmark we now provide the basic properties of this simple static model. This analysis is also useful because it directly applies to the choice problems of the infected and the recovered households in the full model, who structurally solve the same static decision problems. The only truly dynamic decisions are made by susceptible households, whose choices influence their future health status.

To simplify notation, we drop country superscripts and time subscripts for the static analysis of households of country k. Denote the wage by w.

The representative consumer of country k (who is not concerned with health) chooses per-period consumption and labor $(c_k, c_{-k}, \ell) \geq 0$ in order to

$$\max v(x) - \frac{1}{2}\kappa\ell^2$$

$$\text{subject to} \quad x = q(c_k, c_{-k}) \tag{46}$$

$$\widehat{p}_k c_k + \widehat{p}_{-k} c_{-k} = w\ell + g \tag{47}$$

where $\widehat{p}_j$ are consumer prices and g is the public transfer. Let λ denote the Lagrange multiplier of the budget constraint. Importantly, λ measures the pre-epidemic willingness to pay for utility, i.e. the “exchange rate between utils and dollars", which is needed to calibrate the model. As noted in Section 2, the solution is characterized by the following first-order constraints:

$$x^{-\rho}\frac{\partial x}{\partial c_k} = \lambda\widehat{p}_k \tag{48}$$

$$x^{-\rho}\frac{\partial x}{\partial c_{-k}} = \lambda\widehat{p}_{-k} \tag{49}$$

$$\kappa\ell = \lambda w \tag{50}$$

Dividing (48) by (49) yields

$$c_{-k} = \left(\frac{1-\alpha}{\alpha}\right)^{\sigma}\left(\frac{\widehat{p}_k}{\widehat{p}_{-k}}\right)^{\sigma} c_k \tag{51}$$

Hence, unsurprisingly, c_k and c_{-k} are linear functions of each other.

Inserting (51) into (46) yields

$$x = \psi^{\frac{\sigma}{\sigma-1}}\left(\alpha\widehat{p}_{-k}\right)^{-\sigma} c_k \tag{52}$$

where

$$\psi = \alpha^{\sigma}\widehat{p}_{-k}^{\sigma-1} + (1-\alpha)^{\sigma}\widehat{p}_k^{\sigma-1}$$

Inserting (52) into (48), using (50), yields

$$w\psi^{-\frac{\sigma\rho-1}{\sigma-1}}\left(\alpha\widehat{p}_{-k}\right)^{\sigma\rho} c_k^{-\rho} = \kappa\widehat{p}_k\widehat{p}_{-k}\ell \tag{53}$$

By straightworward calculations, the three equations (47), (51), and (53) yield the following solutions for the three unknowns (c_k, c_{-k}, ℓ). Labor ℓ is given by

$$\ell\left(w\ell+g\right)^{\rho} = \frac{w}{\kappa}\psi^{\frac{1-\rho}{\sigma-1}}\left(\widehat{p}_k\widehat{p}_{-k}\right)^{\rho-1} \tag{54}$$

home consumption c_k by

$$\psi\left(\widehat{p}_k\widehat{p}_{-k}\right)^2 c_k^{\rho+1} - \widehat{p}_k\widehat{p}_{-k}\left(\alpha\widehat{p}_{-k}\right)^{\sigma} g c_k^{\rho} = \frac{w^2}{\kappa}\psi^{-\frac{\sigma\rho-1}{\sigma-1}}\left(\alpha\widehat{p}_{-k}\right)^{\sigma(\rho+1)} \tag{55}$$

and foreign consumption by (51). It is easy to see that (54) and (55) each have a unique positive root. Hence, the household problem has a unique solution.

For the case $\rho = 1$, which we use in the numerical calibration, things are particular simple, as both equations are quadratic. In particular, we have

$$\ell = -\frac{g}{2w} + \frac{1}{2w}\sqrt{g^2 + \frac{4w^2}{\kappa}} \tag{56}$$

which yields the multiplier λ, the “price of utility", by (50), as $\lambda = \frac{\kappa}{w}\ell$.

Optimal domestic consumption is

$$c_k = \frac{g\left(\alpha\widehat{p}_{-k}\right)^{\sigma}}{2\psi\widehat{p}_k\widehat{p}_{-k}} + \frac{\left(\alpha\widehat{p}_{-k}\right)^{\sigma}}{2\psi\widehat{p}_k\widehat{p}_{-k}}\sqrt{g^2 + \frac{4w^2}{\kappa}} \tag{57}$$

and foreign consumption correspondingly.

The above analysis describes the demand side of each of the two economies in the absence of health concerns.

A.1.1 No-Pandemic Equilibria

We re-introduce country superscripts to describe market clearing in economies with no health concerns, be it pre-pandemic or after the arrival of a vaccine. The conditions are

$$w^k = p_k z^k \tag{58}$$

$$z^k\ell^k = c_k^k + c_k^{-k} \tag{59}$$

$k = A, B$, for labor market and product market clearing, respectively.

Social Planner Under a benevolent social planner, government policy in each country will be $(\mu^k, \nu^k) = (0, 0)$: levying taxes on domestic or foreign goods is welfare reducing. Hence, the government collects no taxes, and by the budget constraint (26) transfers are $g = 0$. Consumer prices are undistorted,

$$\widehat{p}_k^k = p_k, \widehat{p}_{-k}^k = p_{-k}$$

and the 4 equations (58) and (59) to are sufficient to determine the 4 prices w^k, p_k, $k = A, B$, by using the solutions of (54), (55), and (51) obtained above. Of course, prices are determined only up to one degree of freedom, and by Walras' Law one of the above equilibrium relations is redundant.

Nash In Nash Equilibrium, $\mu^k = 0$ in each country. Yet, tariffs can be positive, for the standard economic reasons of trade wars discussed more broadly in the main text. Hence, consumer prices are

$$\begin{aligned} \widehat{p}_k^k &= p_k \\ \widehat{p}_{-k}^k &= (1 + v^k) p_{-k} \end{aligned}$$

Public transfers are therefore endogenous even in the static setting,

$$g^k = \nu^k p_{-k} c_{-k}^k \tag{60}$$

Now, for given government policies (ν^A, ν^B), we have the 6 equations (58), (59), and (60) to determine the 6 endogenous variables w^k, p_k, g^k, $k = A, B$.

A.1.2 Demand by Infected or Recovered Households

As noted above, the demand of infected and of recovered households in the full model in Section 2 derives from an essentially static optimization problem. Hence, by letting $w = \phi \overline{w}_t^k$ for the infected households of country k at date t, the household optimization conditions of the full model yield the conditions (54), (55), and (51), appropriately indexed for the i households. Similarly, by letting $w = \overline{w}_t^k$ for the recovered households, the household optimization conditions of the full model lead to (54), (55), and (51), appropriately indexed for the r households.

A.2 Disease Transmission

This subsection provides a more detailed microfoundation for the disease transmission dynamics (10) in Section 1.2.

In the basic SIR model (without economic choices) transmission occurs according to

$$T_t = \eta S_t I_t \tag{61}$$

This has the following logic. Let N be size of a given population. Let $N = S+I+R$, where I is the number of infectious, and S that of susceptibles. Let φN be the rate of contacts of a single individual during which the disease can potentially be transmitted.[24] The assumption is that individuals spend a fixed proportion of their time (normalized to 1) outside the home, where they can transmit or contract the virus. Letting θ denote the probability that a contact leads to an infection, equation (61) can now be derived as follows.[25] One susceptible individual outside his home, per unit of time, on average has φN contacts. This leads to $\varphi N(I/N) = \varphi I$ contacts with infectious individuals. The probability of getting infected in these $k = \varphi I$ contacts is

$$\overline{\tau} = 1 - (1-\theta)^k = \theta \sum_{m=0}^{k-1} \binom{k}{m+1} (-\theta)^m \tag{62}$$

for $k > 0$, and the expected total number of transmissions per unit of time is $\overline{\tau} S$. $\overline{\tau}$ as a function of θ is a polynomial of degree k and strictly concave for $k > 1$. Hence, for small θ and large k, $\overline{\tau}$ is smaller than, but approximately equal to $k\theta$. In this case, letting $\eta = \theta\varphi$, the average rate of transmission is approximately equal to

$$\theta k S = \theta \varphi I S = \eta I S$$

as stated in (61).

A.2.1 The Macro-SIR Model

Eichenbaum et al. (2020) have incorporated economic activity into the above model, by distinguishing transmissions while consuming, at work, and during other activities outside the home. This model does not distinguish between foreign and domestic consumption goods.

To make that precise, dropping the time index for convenience, suppose that individuals spend a fixed fraction $f < 1$ of their time outside neither at work nor consuming. All durations

[24] This is the so-called "mass incidence" model which is relevant for Covid-19 (differently from, say, HIV, as analyzed in Greenwood et al. (2019)): one infectious individual can infect a whole (sub-)group, no need for bilateral interaction.

[25] This is the perspective of susceptibles, which is most relevant for economic incentives. Usually, the derivation takes the perspective of infectious. See standard textbooks such as Brauer (2008).

are in terms of the unit of time chosen (which is scaled by φ).[26] To simplify, and different from Eichenbaum, Rebelo and Trabandt (2020); Brotherhood et al. (2020), we do not distinguish between utility from different types of leisure. Hence, individuals do not derive specific utility from leisure outside the home, and we therefore assume this fraction to be constant.[27] Suppose that individuals of health status h spend a fraction $\ell(h) < 1$ of their time at work, and a fraction $\gamma c(h) < 1$ consuming (shopping, dining, ...), the assumption being that the time spent on consumption is proportional to the quantity bought. We assume that $f + \ell(h) + \gamma c(h) < 1$, the remaining time being leisure alone at home.[28] Then, using the linear approximation of the infection probability $\overline{\tau}$, we have the following infection probabilities for susceptibles and aggregate average transmission rates:

1. During non-work-non-consumption time outside the home,
   - individual proba of becoming infected: $f^2\eta I$
   - expected total number of transmissions: $f^2\eta IS$
2. During work,
   - average rate of susceptible contacts with infected per unit of time: $\varphi\ell(i)I$
   - individual proba of becoming infected when working: $\ell(s)\eta\ell(i)I$
   - expected total number of transmissions at work: $\eta\ell(s)\ell(i)IS$
3. During consumption,
   - average rate of contacts with infected per unit of time: $\varphi\gamma c(i)I$
   - individual proba of becoming infected when consuming $c(s)$: $\gamma c(s)\eta\gamma c(i)I$
   - expected total number of transmissions from consumption: $\eta\gamma^2 c(s)c(i)IS$

Hence, an s individual faces the following transition probability to the infected state, if she chooses individual consumption $c(s)$ and labor supply $\ell(s)$:

$$\tau(c(s), \ell(s)) = f^2\eta I + \ell(s)\eta\ell(i)I + c(s)\eta\gamma^2 c(i)I \tag{63}$$

$$= \eta\left[\gamma^2 c(s)c(i) + \ell(s)\ell(i) + f\right] I \tag{64}$$

This yields the expected total number of transmissions from all activities, now with time indices:

$$T_t = \eta\left(\gamma^2 c_t(s)c_t(i) + \ell_t(s)\ell_t(i) + f\right) I_t S_t \tag{65}$$

[26]If this unit is a week and a day has 16 useful hours (e.g. McGrattan, Rogerson et al., 2004), then the individual has $112f$ hours of non-shopping leisure per week outside the home.

[27]See Garibaldi, Moen and Pissarides (2020) for work that endogenizes f in a model of occupational choice, abstracting from the work-consumption choice considered here.

[28]We calibrate the parameter values such that the individual time constraints are satisfied in our simulations. Hence, we can ignore the time constraint in the household's optimization problem of (34).

A.2.2 International transmission

Again dropping the time index for convenience, suppose individuals of country k and health status h spend a fraction $\ell^k(h)$ of their time at work, a fraction $\gamma c_k^k(h)$ of their time consuming the domestic good, a fraction $\gamma c_{-k}^k(h)$ consuming the foreign good, and a fraction f out of their home for other reasons. When "shopping", an individual is directly exposed to home residents and foreigners. Since the contact intensity for foreign and domestic consumption is likely to differ we assume that the consumer has a contact rate $\varphi^d\gamma(C_k^k + C_{-k}^k)$ with domestic residents and a contact rate $\varphi^f\gamma(C_k^{-k} + C_{-k}^{-k})$ with foreigners. In fact, when consuming the domestic good, an individual in country k meets foreign consumers who consume her domestic good, which leads to a number of contacts per unit of time of $\varphi^f\gamma C_k^{-k}$. And when consuming the foreign good, she meets foreign consumers who consume this good, i.e. their domestic good, which leads to a number of contacts per unit of time of $\varphi^f\gamma C_{-k}^{-k}$. Since the consumption of foreign goods is often intermediated by specialized import/export agents and thus likely to involve fewer direct contacts, we expect $\varphi^f < \varphi^d$.

We ignore international encounters at work and in non-work-non-consumption situations. Hence, the transmission dynamics is unchanged from the previous subsection as regards these two types of encounters. With respect to consumption related transmissions, a susceptible consuming the bundle $(c_k^k(s), c_{-k}^k(s))$ has an average rate of contacts with infected per unit of time of

$$\gamma\varphi^d(c_k^k(i) + c_{-k}^k(i))I^k + \gamma\varphi^f(c_k^{-k}(i) + c_{-k}^{-k}(i))I^{-k}$$

where $\gamma\varphi^d c_x^k(i)I^k$ are the contacts with domestic infected and $\gamma\varphi^f c_x^{-k}(i)I^{-k}$ those with foreign infected individuals.

Hence, her individual proba of becoming infected through consumption is approximately

$$c_k^k(s)\theta\gamma^2\left[\varphi^d c_k^k(i)I^k + \varphi^f c_k^{-k}(i)I^{-k}\right] + c_{-k}^k(s)\theta\gamma^2\left[\varphi^d c_{-k}^k(i)I^k + \varphi^f c_{-k}^{-k}(i)I^{-k}\right]$$

Adding the infection probabilities yields the formulas (9) and (10) in the main text. These transmission dynamics are the simplest possible generalization of those of the single good case (65). The new terms reflect the transmissions through consumption interactions in exports ($c_{kt}^{-k}(i)$) and imports ($c_{-kt}^k(i)$) and therefore also involve foreign consumption abroad, $c_{-kt}^{-k}(i)$. More complicated interaction models (interactions at work or between consumption and leisure) do not change the results significantly.

A.3 Computation Details

The numerical algorithm for solving our model proceeds in a number of steps. We first detail the solution to the model for fixed containment policies and then detail the solution for the optimal coordinated and uncoordinated policies.

Solution for fixed policies. To solve the model for a fixed set of containment taxes, we begin with guesses for the susceptible households' labor and consumption choices in each country and period as well as the relative price of country B's good in each period. Note that we normalize country A prices to 1. Given these guesses, we calculate the implied government tax as well as the labor and consumption of all other household types. We then iterate forward on the SIR equations until the final period of the model, at which point consumption and labor return to their steady state values due to the vaccine's arrival. Next, we iterate backward to derive the present value of lifetime utility for each agent. We then use gradient-based methods to adjust our initial guesses until the susceptible agents' first-order conditions, market clearing conditions, and government budget constraints hold. In this way, we confirm all equilibrium conditions are satisfied.

Social planner solution. To solve for optimal containment policies from the perspective of a social planner, we nest the solution for fixed policies within another gradient-based optimizer. In this outer loop, we solve for containment policies and tariffs which maximize the present value of total time-0 utility, equally weighted across both countries.

Nash equilibrium solution. To solve for the Nash Equilibrium containment policies we begin with a guess for containment policies and tariffs across both countries. Given a fixed policy for a given country, we use a gradient-based optimizer to find the optimal policy response of the other country that maximizes the welfare of its own households. We then take this policy as fixed and find the optimal policy response of the other country. We iterate on this procedure until both countries' policies are the best responses to each other. We experiment with many different starting values but do not find any differences in the final result, which makes us believe that the identified Nash equilibrium is unique.