The Macroeconomics of Age-Varying Epidemics*

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Abstract

We incorporate age-specific socio-economic interactions in a SIR macroeconomic model to study the role of demographic factors for the COVID-19 epidemic evolution, its macroeconomic effects and possible containment measures. We capture the endogenous response of rational individuals who freely reduce consumption- and labor-related personal exposure to the virus, with interactions that can vary within and across ages, while fail to internalize the impact of their actions on others. The endogenous response amplifies the economic losses, but it also implies that the individual behavioral response to the risk of infection is an important ally of the needed policy measures to contain the spread of the virus. Investigating the effect of different combinations of economic shutdown and age-targeted social distancing we find that there are considerable economic benefits from measures targeting the elderly with higher mortality risk which are not part of the labor force. For any level of social distancing, the implied optimal economic shutdown generates small gains in terms of lives and large output losses over one-year time. These results are confirmed by calibrating the model to match real epidemic and economic data in the context of a scenarios exercise.

JEL codes: E1, I1, H0

Keywords: Epidemic, COVID-19, recessions, demographics, age differences, vaccine, containment policies, SIR macro model

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1 Introduction

Older people are more vulnerable to COVID-19 as shown in an increasing number of studies.[1] In addition, countries with many intergenerational contacts may see faster transmissions to elderly (Dowd et al., 2020) as indicated by the positive correlation across countries between the case fatality rate and the share of 30 to 49 year-old people living with their parents (Kuhn and Bayer, 2020). Containing the overall death toll of the disease could come from controlling the number of potential infectious contacts of those with a higher probability of dying.

Merging the epidemiological SIR models *à la* Kermack and McKendrick (1927) with macroeconomic models, Eichenbaum et al. (2020b) (ERT, henceforth) developed a "SIR macro" model. This model reveals that susceptible rational agents severely reduce their consumption and hours worked to lower their own probability of getting infected.[2] Meanwhile, they do not internalize their impact on the overall spread of the infection. To contain this externality, public policy measures are necessary.

As noted in Ellison (2020), allowing for variation in the contact rates by considering how sub-populations differ in their activity levels is a primary objective. We contribute via combining intergenerational and economic interactions developing a "SIR-age macro" model. Our reference model are ERT and Towers and Feng (2012) which we extend via allowing for age-specific economic interactions. We evaluate how this affects macroeconomic outcomes, and how these in turn depend on the design of age-specific containment policies.

We consider two age-groups: aged 70 or more (elderly) and the rest of the population (young). Susceptible agents can get infected. If infected, the individual either recovers (and cannot get infected again) or dies. The elderly face a higher mortality risk,[3] they do not work and consume their fixed

[1]See Dudel et al. (2020); Goldstein and Lee (2020) for evidence.

[2]See Farboodi et al. (2020) for the United States, Andersen et al. (2020) for Denmark and Sweden.

[3]Data on the South Korean fatality rates (supposedly the most reliable, given the world's highest per capita test rates for COVID-19) and estimates of the infection fatality rates for Italy (Garibaldi et al., 2020), that we both use in our analysis, give that those aged 70 or more face a mortality probability upon infection of 22 to 28 times higher than the rest of the population. However, uncertainty remains on the actual level of the mortality risk (Goldstein and Lee, 2020).

pension. All individuals are hand-to-mouth. Interactions occur when consuming, working or for other residual activities. Each type of interaction has a different infection probability which, among other parameters, depends on the exogenous daily number of contacts (between and within age-groups).

The pandemic can be contained either, by means of a consumption tax controlled by the government (as in ERT), referred to as "economic shutdown", or alternatively, by "social distancing", reducing the number of interactions.

While the latter does not directly impair economic activity, the former makes consumption and hence production more expensive. In our model, social distancing can be attributed not only to mandated policies, but also to changes in behavior by individuals other than consumption and labor choices (e.g. the use of personal protective equipment).[4]

The no containment scenario is characterized by the highest death toll (0.82% of the population) and an output loss the first year of 3.86% as compared to the pre-epidemic steady state. Using a generalized social distancing lowers the corresponding output loss (1.47%) and death toll (0.21% of the population). Deviating from generalized social distancing via milder restriction on young interactions, results in higher deaths (0.47%) and higher output loss (2.85%). Via instead letting the elderly interact less with the young implies a 1.9% output loss (worse than generalized social distancing, but better than mild social distancing for young) and 0.11% deaths (better than generalized and mild on young distancing).

Without social distancing, an optimal economic shutdown is characterized by 0.43% deaths and 30.1% yearly output losses. Given different social distancing measures, the optimal economic shutdown generates 19.8%, 24.9% and 9.6% yearly output losses in the case of generalized social distancing, milder on young and stricter on old respectively. For any level of social distancing, the implied optimal economic shutdown generates small gains in terms of lives and large increases in terms of yearly output losses. To minimize output and death losses one should search for the policy mix that

[4]Cf. Baqaee et al. (2020) who also consider reduction in contacts both reflecting behavioral and mandated changes.

flattens the infection curve of the elderly the most, rather than the overall infection curve.

We evaluate the model's capacity of replicating actual data using Italy as one of the countries firstly affected by the epidemic at the global level. We calibrate our model, including the containment policies, to target the number of excessive deaths (as obtained in Galeotti et al. (2020)) and to capture the 5.3% quarterly output loss in the first quarter of 2020.[5] The simulations suggest that about three weeks were necessary to flatten the curve of total deaths due to COVID-19 and that absent any government intervention more than 0.4% of the population would have died in two months. Given this epidemic evolution in line with the data, we evaluate the epidemic and its economic impact in a series of hypothetical post-lockdown scenarios corresponding to different types of social distancing for any given economic shutdown.

We recognize the uncertainty surrounding many of the model parameters. Our quantitative results are better interpreted in terms of a relative order of magnitude within the internal modeling consistency. Our model points to the preference of differentiating containment measures by age, acting more on social distancing of the elderly from the young than on homogeneous economic measures. Finally, while the recommendations stemming from the model points to the benefits of reducing the number of contacts between the elderly and the young, there are clearly human costs associated with long-term isolation that we are not considering.

This work contributes to the fast increasing literature on the economic consequences of the COVID-19 epidemic. The contributions in economic modeling has been ranging from *purely epidemiological models* (Acemoglu et al., 2020; Alvarez et al., 2020; Atkeson, 2020a,b; Berger et al., 2020; Chikina and Pegden, 2020; Favero et al., 2020; Fernndez-Villaverde and Jones, 2020; Rampini, 2020; Stock, 2020); [6] through *epidemiological models with choices of rational economic agents* other than the social planner (Bodenstein et al., 2020; Brotherhood et al., 2020; Eichenbaum et al., 2020b,c;

[5] ISTAT (2020)

[6] Criticisms to the use of SIR models for policy evaluation are offered by Chang and Velasco (2020) and Von Thadden (2020).

Farboodi et al., 2020; Garibaldi et al., 2020; Glover et al., 2020; Jones et al., 2020; Kapička and Rupert, 2020; Kaplan et al., 2020; Krueger et al., 2020); to *purely economic models* (Faria-e-Castro, 2020; Gregory et al., 2020; Guerrieri et al., 2020; McKibbin and Fernando, 2020).

Other work considers age heterogeneity with respect to the COVID-19 epidemics. We differ from Glover et al. (2020) as, while not considering redistributive aspects, we allow for the likelihood of infection to increase with consumption focusing on the role of age-specific containment policies for aggregate health-output trade-offs. Compared to Brotherhood et al. (2020), we explicitly consider the empirical intergenerational contacts that prevail in "normal times", tailoring our model to the Italian case for policy scenarios. Besides the revision of individual choices on consumption and labor to reduce the exposure for a given number of daily normal contacts, it might well be that individuals freely revise their number of contacts to a "new normal" for a while. This could be captured in our model by an exogenous reduction in the number of daily contacts and would therefore reduce the need of regulation.

We also extend on Acemoglu et al. (2020), Chikina and Pegden (2020), Favero et al. (2020), Gollier (2020) and Rampini (2020) via specifically modeling age-specific interactions as economic ones capturing the endogenous consumption/labor response to the epidemic progression.

The rest of the paper is organized as follows. Section 2 describes the model. Section 3 details the main calibration of the model and the *laissez-faire* equilibrium in comparison also to ERT. Section 4 studies social distancing and the optimal economic shutdown, first separately and then as a policy mix. Section 5 applies the model to the Italian case to study different scenarios. Section 6 concludes.

2 The age-varying SIR macro model

To a purely epidemiological age-varying SIR model, building on Eichenbaum et al. (2020b),[7] we add macroeconomic interactions affecting the number of infected people.

2.1 The age-varying SIR model with macroeconomic interactions

The population is grouped in two categories: young (y) and old (o). For age $a \in \{y, o\}$ and discrete time t the following set of equations holds:

$$
\begin{aligned}
S_{a,t+1} &= S_{a,t} - T_{a,t} && (2.1)\\
I_{a,t+1} &= I_{a,t} + T_{a,t} - (\pi_{a,r} + \pi_{a,d}) I_{a,t} && (2.2)\\
R_{a,t+1} &= R_{a,t} + \pi_{a,r} I_{a,t} && (2.3)\\
D_{a,t+1} &= D_{a,t} + \pi_{a,d} I_{a,t} && (2.4)\\
N_{a,t} &= S_{a,t} + I_{a,t} + R_{a,t} && (2.5)\\
N_t &= \sum_a N_{a,t} && (2.6)
\end{aligned}
$$

The number of newly infected people in each period in each category is given by:

$$
\begin{aligned}
T_{y,t} = \eta S_{y,t} \Bigg[&\pi^s_{y,1} z_{y,y} \frac{I_{y,t}}{f_y} c^i_{y,t} c^s_{y,t} + \pi^s_{y,2} z_{y,o} \frac{I_{o,t}}{f_o} c^i_{o,t} c^s_{y,t} + \pi^s_{y,3} z_{y,y} \frac{I_{y,t}}{f_y} n^i_{y,t} n^s_{y,t} \\
&+ \pi^s_{y,4} \left(z_{y,y} \frac{I_{y,t}}{f_y} + z_{y,o} \frac{I_{o,t}}{f_o} \right) \Bigg] && (2.7)
\end{aligned}
$$

$$
T_{o,t} = \eta S_{o,t} \left[\pi^s_{o,1} z_{o,o} \frac{I_{o,t}}{f_o} c^i_{o,t} c^s_{o,t} + \pi^s_{o,2} z_{o,y} \frac{I_{y,t}}{f_y} c^i_{y,t} c^s_{o,t} + \pi^s_{o,3} \left(z_{o,y} \frac{I_{y,t}}{f_y} + z_{o,o} \frac{I_{o,t}}{f_o} \right) \right] \qquad (2.8)
$$

where $z_{\cdot,\cdot}$ denotes the elements of the contact matrix:

$$
Z = \begin{bmatrix} z_{y,y} & z_{y,o} \\ z_{o,y} & z_{o,o} \end{bmatrix} \qquad (2.9)
$$

[7] If we shut down the possibility of economic interactions the model described in the first subsection becomes a standard age-varying SIR model as employed in the epidemiological literature (see e.g. Towers and Feng (2012)).

describing the number of contacts between and within age groups.[8] Denoting by f_o and $f_y = 1-f_o$ the initial fraction of old and young respectively, this matrix must satisfy: $f_o z_{o,y} = (1-f_o)z_{y,o}$. Each $\pi^s_{\cdot,\cdot}$ parameter captures the weight to each type of infectious interaction.[9]

The initial population is normalized to one, $N_0 = 1$. We assume that there is an initial shock ε to the total number of infected across the age groups according to: $I_{o,0} = \varepsilon f_o$ and $I_{y,0} = \varepsilon - I_{o,0}$.

2.1.1 Agents' choices in the macroeconomy

The budget constraints for the young and the old individuals for $j \in \{s, i, r\}$ are:

$$\begin{aligned}(1+\mu_{c,t})c^j_{y,t} &= w_t \phi^j n^j_{y,t} + \Gamma_t, \quad \phi^s = \phi^r = 1, \quad \phi^i < 1 \\ (1+\mu_{c,t})c^j_{o,t} &= \bar{P} + \Gamma_t\end{aligned}$$

The elderly receive a constant pension transfer proportional to the steady state consumption of the young: $\bar{P} = \alpha c^s$, $0 < \alpha < 1$.[10] The government can set the consumption tax $\mu_{c,t}$ and distribute lump-sum transfers Γ_t according to the following budget constraint:

$$\mu_{c,t} C_t = \Gamma_t N_t \tag{2.10}$$

where

$$C_t = c^s_{y,t} S_{y,t} + c^i_{y,t} I_{y,t} + c^r_{y,t} R_{y,t} + c^s_{o,t} S_{o,t} + c^i_{o,t} I_{o,t} + c^r_{o,t} R_{o,t} \tag{2.11}$$

We assume the following utility function:

$$u(c,n) = \log c - \frac{\theta}{2} n^2$$

According to their type – young or old who are either susceptible, infected or recovered – individuals satisfy the following dynamic programming.

[8] In this simple case, for example, the first raw of Z denotes the number of contacts per period that a young makes with a young ($z_{y,y}$) and with an old ($z_{y,o}$).

[9] Setting $\pi^s_{y,1} = \pi^s_{y,2} = \pi^s_{y,3} = \pi^s_{o,1} = \pi^s_{o,2} = 0$ and $\pi^s_{y,4} = \pi^s_{o,3} = 1$ the model is a epidemiological age-varying SIR model without economic interactions. The ERT model without age variations is nested assuming that $z_{y,o} = z_{o,y} = z_{o,o} = 0$ and $f_y = 1$.

[10] Since our focus is on the short-run that pertains the outbreak of an epidemic we abstract from long-run issues such as transfers among individuals of different age classes as well as public debt sustainability.

Susceptibles.

$$U^s_{y,t} = u(c^s_{y,t}, n^s_{y,t}) + \beta\left[(1-\tau_{y,t})U^s_{y,t+1} + \tau_{y,t}U^i_{y,t+1}\right](1-\delta_v) + \delta_v \beta U^r_{y,t+1} \tag{2.12}$$

$$U^s_{o,t} = u(c^s_{o,t}, 0) + \beta\left[(1-\tau_{o,t})U^s_{y,t+1} + \tau_{o,t}U^i_{o,t+1}\right](1-\delta_v) + \delta_v \beta U^r_{o,t+1} \tag{2.13}$$

$$\tau_{y,t} = \frac{T_{y,t}}{S_{y,t}} \tag{2.14}$$

$$\tau_{o,t} = \frac{T_{o,t}}{S_{o,t}} \tag{2.15}$$

where δ_v is the per period probability of discovering a vaccine against the virus. As in ERT we assume that upon discovery the vaccine is administered to all the susceptibles in the country from the period of the discovery. Once a person is vaccinated this person becomes immune to the disease.

Infected.

$$U^i_{y,t} = u(c^i_{y,t}, n^i_{y,t}) + \beta\left[(1-\pi_{y,d}-\pi_{y,r})U^i_{y,t+1} + \pi_{y,r}U^r_{y,t+1}\right](1-\delta_c) + \delta_c \beta U^r_{y,t+1} \tag{2.16}$$

$$U^i_{o,t} = u(c^i_{o,t}, 0) + \beta\left[(1-\pi_{o,d}-\pi_{o,r})U^i_{o,t+1} + \pi_{o,r}U^r_{o,t+1}\right](1-\delta_c) + \delta_c \beta U^r_{o,t+1} \tag{2.17}$$

Recovered.

$$U^r_{y,t} = u(c^r_{y,t}, n^r_{y,t}) + \beta U^r_{y,t+1} \tag{2.18}$$

$$U^r_{o,t} = u(c^r_{o,t}, 0) + \beta U^r_{o,t+1} \tag{2.19}$$

Firms. There is a continuum of competitive representative firms of unit measure that produce consumption goods (Y_t) using hours worked (H_t) according to the technology $Y_t = AH_t$ to maximize profit:

$$\max_{H_t} \{AH_t - w_t H_t\}$$

which leads to the optimal condition: $w_t = A$.

Clearing.

$$C_t = AH_t + \bar{P}N_{o,t} \tag{2.20}$$

$$n^s_{y,t}S_{y,t} + \phi^i n^i_{y,t}I_{y,t} + n^r_{y,t}R_{y,t} = H_t \tag{2.21}$$

Welfare. When computing optimality of policy interventions we assume the following aggregate welfare in the first period of the epidemic:

$$U_0 = S_{o,0}U^s_{o,0} + I_{o,0}U^i_{o,0} + S_{y,0}U^s_{y,0} + I_{y,0}U^i_{y,0} \tag{2.22}$$

where the terms $U^s_{\cdot,0}$ and $U^i_{\cdot,0}$ represent the lifetime utilities of susceptibles and infected agents in each age groups.

3 *Laissez-faire* equilibrium

3.1 Contact matrix & parameter values

The initial share of young (those younger than 70) is set to $f_y = 0.825$.[11]

The contact matrix Z in equation (2.9) is based on values from Mossong et al. (2017) contact survey data[12] and widely used in the epidemiological literature.[13] The young have on average about 19.1 contacts per day with individuals of the same age and 1.3 contacts per day with the older group. The elderly have on average 6.3 contacts per day with the young and 1.4 contacts per day with other elderly.[14] Notably, young individuals have more contacts and there is a tendency for within age group interactions.[15]

In the initial pre-infection steady state the population is composed only by susceptible individ-

[11] Compatible with the shares for Italy United Nations World Population Prospects 2019

[12] Dataset available via the package `socialmixr`. The main reference is Mossong et al. (2008). A contact is defined as "either skin-to-skin contact such as a kiss or handshake (a physical contact), or a two-way conversation with three or more words in the physical presence of another person but no skin-to-skin contact (a nonphysical contact)".

[13] See e.g. Towers and Feng (2012) and in the COVID-19 related epidemiological literature Ferguson et al. (2020) and Figure 7 in Appendix C

[14] The contact matrix needs to respect a symmetric property such that $f_o z_{o,y} = f_y z_{y,o}$ which implies: $z_{o,y} = [f_y/(1 - f_y)]z_{y,o} = [0.825/(1 - 0.825)]1.337 = 6.303$.

[15] These patterns are confirmed in Appendix D where we show the contact matrix for the whole sample in the Mossong et al. (2017) survey data (namely, including also Germany, Luxembourg, Netherlands, Poland, United Kingdom, Finland, Belgium).

uals which yields:

$$n_y^s = \theta^{-0.5}$$

$$c_y^s = wn_y^s = A\theta^{-0.5}$$

$$c_o^s = \bar{P} = \alpha c_y^s = \alpha A\theta^{-0.5}$$

We assume $\alpha = 0.8$, i.e. the steady state consumption of an old individual is 80% of the steady state consumption of a young.[16] Following Towers and Feng (2012), we assume that the "removal" rate γ (the rate at which an infected individual either recovers or dies) is equal across age classes. Similarly to ERT and Atkeson (2020b), we assume it takes 18 days to either recover or die for both ages, $\gamma = 7/18$.

Considering the case fatality rates by age reported by South Korean Ministry of Health and Welfare on April 5, 2020[17] and interacting them with the demographic Italian shares in year 2019[18] we set the ratio between the probability of dying and the removal rate to be 0.0045 and 0.1273 for the young and the old respectively.

We calibrate the parameter η in equations (2.7)–(2.8) relying on the SIR-age model, with no economic interactions. In this case η represents the transmission rate obtained from the expression for the *basic reproduction number* $\mathcal{R}_0$[19] for a SIR-age model from Towers and Feng (2012):

$$\eta = \frac{\gamma \mathcal{R}_0}{\max\{\text{eig}(M)\}} \tag{3.1}$$

where $\max\{\text{eig}(M)\}$ denote the largest eigenvalue of the M matrix:

$$M = \begin{bmatrix} z_{y,y}\frac{f_y}{f_y} & z_{y,o}\frac{f_y}{f_o} \\ z_{o,y}\frac{f_o}{f_y} & z_{y,o}\frac{f_o}{f_o} \end{bmatrix}$$

[16] This number reflects the life-cycle profile of consumption (see e.g. Fernandez-Villaverde and Krueger (2007)) with average consumption during retirement generally smaller than what one has during working-age periods.

[17] See https://www.cdc.go.kr/board/board.es?mid=a30402000000&bid=0030&act=view&list_no=366739&tag=&nPage=1

[18] See https://population.un.org/wpp/Download/Standard/Population/

[19] $\mathcal{R}_0$ is defined as the average number of secondary infections produced by one infected individual during his/her entire period of infection in an entirely susceptible population.

We set the $\mathcal{R}_0$ to 1.59 such that 60% of the initial population either recovers or die[20] implying $\eta = 0.0045$.

To calibrate all the $\pi^s_{\cdot,\cdot}$ parameters we revise the approach in ERT to account for the age-varying nature of our model. We assume that consumption and labor activities account for two-third of all the infection transmissions for each age class i.e. $\pi^s_{\cdot,\cdot}$ satisfy:[21]

$$\frac{\pi^s_{y,1} z_{y,y} (c_y)^2}{\Pi_y} = \frac{\pi^s_{y,2} z_{y,o} c_y c_o}{\Pi_y} = \frac{\pi^s_{y,3} z_{y,y} (n_y)^2}{\Pi_y} = 1/9,$$
$$\frac{\pi^s_{o,1} z_{o,o} (c_o)^2}{\Pi_o} = \frac{\pi^s_{o,2} z_{o,y} c_y c_o}{\Pi_o} = 1/6$$

where

$$\begin{aligned}
\Pi_y &= \pi^s_{y,1} z_{y,y} (c_y)^2 + \pi^s_{y,2} z_{y,o} c_y c_o + \pi^s_{y,3} z_{y,y} (n_y)^2 + \pi^s_{y,4} (z_{y,y} + z_{y,o}) \\
\Pi_o &= \pi^s_{o,1} z_{o,o} (c_o)^2 + \pi^s_{o,2} z_{o,y} c_y c_o + \pi^s_{o,3} (z_{o,y} + z_{o,o})
\end{aligned}$$

A second set of conditions returns limit-values for the number of people who either recover or die at the end of the epidemic:

$$\lim_{t\to\infty} \{R_{y,t} + D_{y,t}\} = 0.545 \qquad\qquad \lim_{t\to\infty} \{R_{o,t} + D_{o,t}\} = 0.055$$

which correspond to targeting $\lim_{t\to\infty}\{R_{y,t} + D_{y,t} + R_{o,t} + D_{o,t}\} = 0.6$.

The remaining parameters are set to the values assumed in ERT. In particular, $A = 39.835$, $\theta = 0.001275$ so that in the pre-epidemic steady state the average working week is 28 hours and the average weekly earnings is $\$58000/52$ US dollars. The discount factor $\beta = 0.96^{1/52}$ implies one life value equals 9.3 million 2019 dollars in the pre-epidemic steady state. The relative

[20] As assumed by ERT and outlined by Angela Merkel in her March 11, 2020 speech https://www.nytimes.com/2020/03/11/world/europe/coronavirus-merkel-germany.html.

[21] Implying values for the shares of initial jump in the transmission probability due to the different causes of infection (consumption, work, residual). Consider for example equations (2.14) and (2.7). At the time of the initial infectious shock, given $I_{y,0} = \varepsilon f_y, I_{o,0} = \varepsilon f_o$, we have:

$$\tau_{y,0} = \varepsilon\eta \left[\pi^s_{y,1} z_{y,y} (c_y)^2 + \pi^s_{y,2} z_{y,o} c_o c_y + \pi^s_{y,3} z_{y,y} (n_y)^2 + \pi^s_{y,4} (z_{y,y} + z_{y,o})\right]$$

productivity of infected people is set to $\phi^i = 0.8$. The fraction of people initially infected ε is set to 0.1%. In the *laissez-faire* equilibrium there is no policy intervention i.e. $\mu_{c,t} = 0$ for all t and the frequency of contacts among individuals is set to the values in Figure 7.

3.2 Results: SIR-age macro model vs SIR-age model

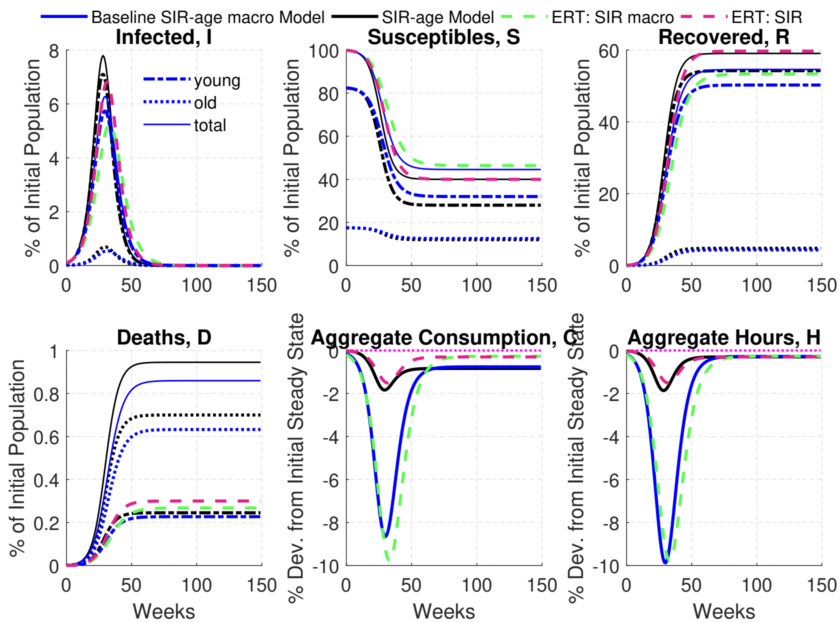

Figure 1: *Laissez-faire* SIR-age macro model vs SIR-age model vs Eichenbaum et al. (2020b) (ERT) models

Note. The SIR-age model is obtained by setting $\pi^s_{y,1} = \pi^s_{y,2} = \pi^s_{y,4} = \pi^s_{o,1} = \pi^s_{o,2} = 0, \pi^s_{y,3} = \pi^s_{o,3} = 1$.

Figure 1 shows three sets of results from different models: (i) the model from Eichenbaum et al. (2020b) (a SIR-macro model with no age differences); (ii) the "SIR-age macro model", a SIR model with two age groups and economic interactions in the infection transmission (2.7)-(2.8), in a *laissez-faire* environment; (iii) the "SIR-age model", a "mechanistic" epidemiological model without the endogenous behavioral response of individuals. In line with ERT, the baseline SIR-macro model predicts fewer deaths and a sharper recession than a model without the feedback from the economy to disease diffusion due to that susceptibles reduce their consumption and hours worked to lower their probability of being infected. At the end of the transition, 0.86% of the initial population dies in the SIR-age macro model versus 0.95% in the SIR-age model (with the elderly representing 0.63% and 0.70% respectively).

These numbers are both considerably larger than the ERT baseline where 0.27% (0.3%) of the initial population dies in their SIR macro (SIR) model as they exclude those older than 70 while we focus explicitly on this age-class.[22]

The recession in the SIR-age macro model is more than four times worse than in the SIR-age model. The average aggregate consumption in the first year of the epidemic falls by 3.86% (0.9%) in the SIR-age macro (SIR-age) model. From the peak-to-trough aggregate consumption decreases by 8.66% (1.84%) respectively. In the long-run aggregate consumption is permanently lowered by the death toll: 0.76% lower in the SIR-age macro model compared to 0.83% in the SIR-age model.[23] In spite of the permanent effect of the death toll, the recession is relatively reabsorbed, with aggregate consumption after one year standing at -1.7% (-0.88%) of its pre-infection level. [24]

4 Containment policies

Based on the SIR-age-macro model we consider different possible ways of containing the epidemics spread: (1) a consumption tax μ_c inducing a reduction in the level of consumption and labor, referred to as "economic shutdown"; (2) the practice of "social distancing" which could be both mandated and behavioral implemented reducing the values of the entries $z_{\cdot,\cdot}$ in the contact matrix. Reductions in these contacts can be attributed not only to enforced measures, but also to behavioral changes (other than consumption and labor) e.g. the use of personal protective equipment.

[22] Recall that we assume that 60% of initial population either recovers of dies in the limit of the SIR-age model. This is also what ERT assume in their SIR model. Hence, with the same ending number of total susceptibles (and roughly with their same evolution, see raw 1, column 2 of Figure 1) we obtain a much bigger death toll than ERT due to the higher case fatality rate of the elderly.

[23] Steady state per capita consumption is given by $C/N = c_y^r[1-(1-\alpha)N_o/N]$. Young individuals consume the same in both the initial and final steady state (c_y^r) and the elderly are given the same fraction of consumption (αc_y^r). Since the infection reduces the proportion of elderly in the economy (N_o/N), per capita consumption in the final steady state will be (slightly) higher than in the initial steady state.

[24] Compared to ERT, our SIR-age macro model predicts a milder recession reflecting the fact that only young individuals work in our economy while the elderly are always granted a certain level of consumption.

4.1 Social distancing

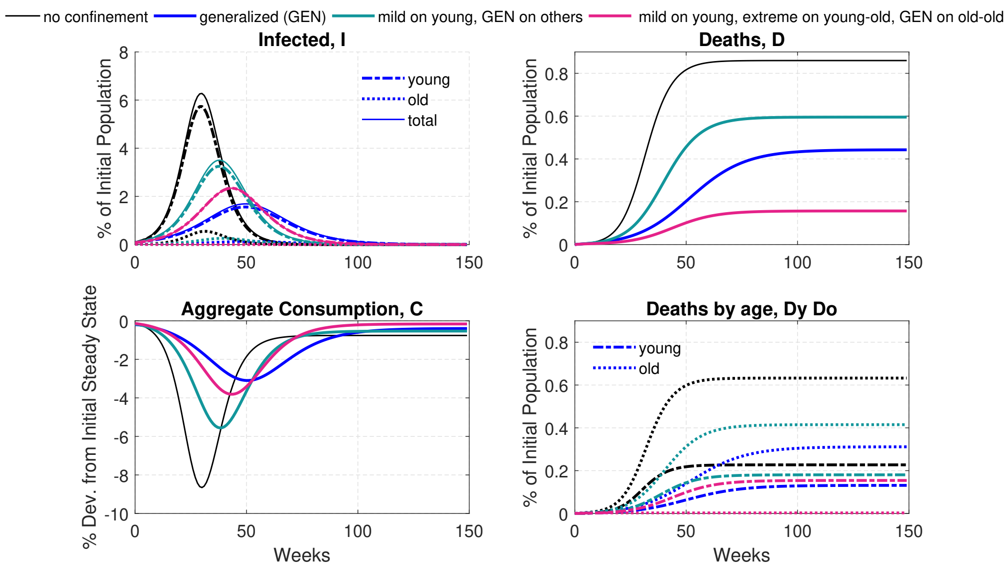

Figure 2: SIR-age macro model: different confinements

Note. The scenario “generalized (GEN)” assumes that each social contact is diminished by 20% with respect to the baseline (“no containment”). The scenario “mild on young, GEN on others” is the same as GEN but the social distancing between young individuals is 50% less severe than in GEN. The scenario “mild on young, extreme on young-old, GEN on old-old” is the same as the previous scenario but assumes that no contact is allowed between young and old individuals.

In Figure 2 we show the effect of different social distancing measures throughout the transition dynamics and we compare these scenarios with the baseline SIR-age macro model results (black lines) i.e. assuming freedom in social contacts.

A generalized policy applied to all individuals (blue lines) modelled as a generalized 20% cut in social contacts flattens the epidemic curve (halved death toll) and lowers aggregate consumption by 3% over the first year prolonging the recession as compared to the baseline. This reflects the smaller death toll (0.44% of the initial population dies versus 0.86% in the baseline) which is disproportionately borne by the old individuals.

The green line in Figure 2 shows the case in which the confinements measures restricting the contacts among young are milder: i.e. social distancing is 50% less severe if it involves contacts among young individuals (while the generalized distancing applies to all other contacts). This implies that contacts among young individuals are 90% of the baseline and it generates a severer epidemic and a sharper recession whereas the elderly bear disproportionately more the brunt of the death toll.

Is it possible to impose a milder social distancing on contacts among young while containing the death toll and the recession imposing a total isolation of the elderly from the young as shown in the violet lines in Figure 2. In this case the death toll is significantly reduced (0.15$\%$ of the initial population dies by the end of the epidemic) with young individuals being relatively more represented in the death toll.

4.2 Shutdown of economic activity

A shutdown of economic activity is implemented (in absence of social distancing) increasing the consumption tax parameter $\mu_{c,t}$. The optimal economic shutdown μ_c is the one that maximizes the expected utility of all agents in the economy, i.e. our welfare function. The optimal level of shutdown is considered both in the case a vaccine is expected within one year from the onset of the epidemic $\delta_v = 1/52$, two years from the beginning of the epidemic $\delta_v = 0.5/52$ or when a vaccine is found with zero probability.

Two forces regulate the optimal shutdown level: (1) we want to minimize the number of deaths (this calls for a higher intensity of shutdown); (2) we want to maximize aggregate consumption (this calls for a lower intensity of the shutdown).

In Figure 3 we can see that when a vaccine is possible, it is optimal to immediately introduce severe containment measures. The closer in time the vaccine is expected to be, the more optimal it is to delay infections as a larger number of susceptible will then benefit from the vaccination (infection peaks in week 42, 45 and 48 when vaccine is available in two years, one year or never respectively). With a vaccine on the horizon the infection peaks at lower percentages of the initial population and the cumulative death toll is slightly higher (about 0.65$\%$ of the initial population versus 0.6$\%$ in the case without vaccine) in line with less restrictive optimal shutdown throughout the epidemic.

Intuitively, knowing that a vaccine will be available tomorrow, it is preferable to postpone

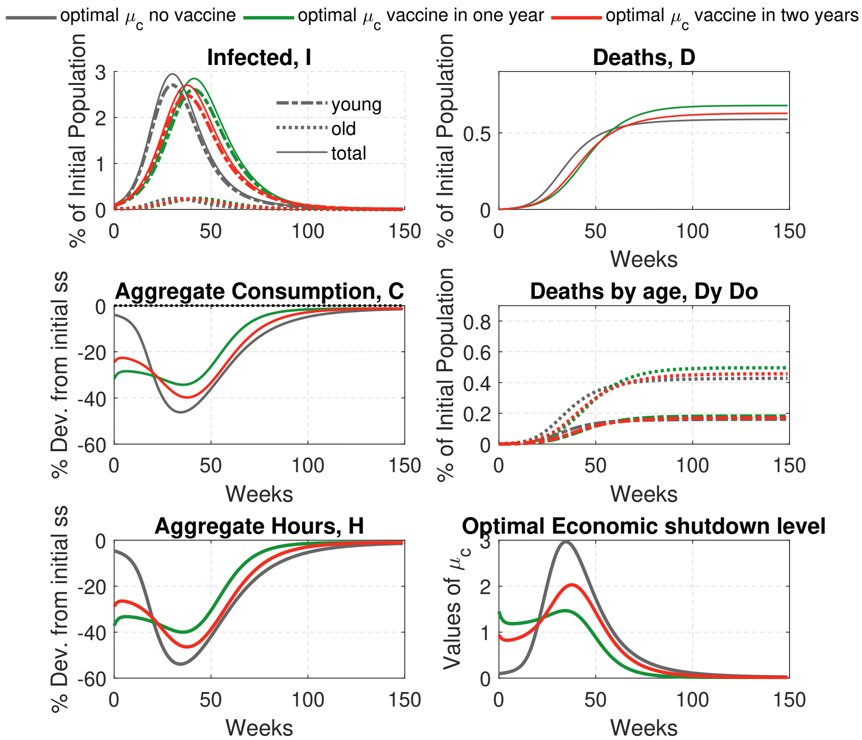

Figure 3: SIR-age macro model: Optimal shutdown by vaccine horizon

consumption until when it is safe, hence the optimal shutdown curve is higher (green). This is associated with a larger initial drop in consumption the sooner the vaccine. It follows that the further away the vaccine is, the higher the immediate pay-off is realized from not constraining the economic activity.

Over time, as the epidemic spreads, the average level of contagion rises and the mortality-diminishing motive prevails over the consumption-rising motive in the utility maximization. This makes it optimal to rise the shutdown level, the more so, the lower the initial shutdown with consequent higher output losses up to 45% with respect to the pre-epidemic. Reducing the epidemic duration and intensity also has an amplifying positive effect for the macroeconomy in our framework as it endogenously encourages consumption.

The annual average output loss amounts to 29.6% and 31% in the case of optimal shutdown with vaccine in one year and in two years respectively, in line with ERT simulations.

4.3 Optimal shutdown with social distancing

The containment policy combination (restricting both the economic activity[25] and the contacts among people) can be proxied by a joint implementation of a uniform economic shutdown and social distancing policies.

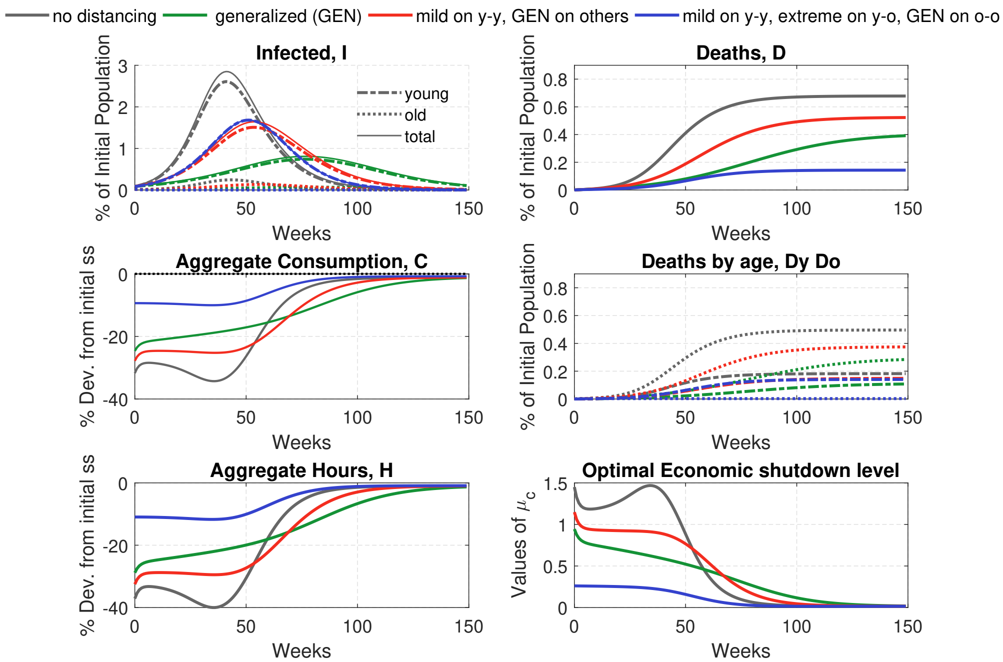

Figure 4: SIR-age macro model: Optimal shutdown $\mu_{c,t}$ with social distancing

Note. The optimal economic shutdown parameter μ_c is computed in the case in which a vaccine is discovered in one-year time from the outbreak of the epidemic for different social distancing scenarios. The scenario "generalized (GEN)" assumes that each social contact is diminished by 20% with respect to the baseline ("no distancing"). The scenario "mild on y-y, GEN on others" is the same as GEN but the social distancing among young individuals is 50% less severe than in GEN. The scenario "mild on y-y, extreme on y-o, GEN on o-o" is the same as the previous scenario but assumes that no contact is allowed between young and old individuals.

As both social distancing and economic shutdown contribute to flattening the infection curve with varying degrees of consumption losses, different social distancing measures of section 4.1 imply optimal different levels of economic shutdown displayed in Figure 4 where we have assumed that a vaccine becomes available on average in one year time after the beginning of the epidemic.[26]

[25] Across a broad set of industries with retail entertainment, restaurants and travel among the most affected.
[26] In Figure 10 in Appendix D we observe the corresponding scenarios in absence of a vaccine possibility.

The policy minimizing consumption and death losses is not the one flattening the overall infection curve, but the policy mix flattening the elderly infections' curve the most.

The social distancing measure minimizing the optimal economic shutdown and the consumption reduction is the isolation of the old from the young population which also delivers the most favorable economic outcome over one year horizon (9.6% losses with respect to the pre-epidemic equilibrium). In this case the old isolate from the more socially active young dramatically cutting the contagion possibilities and therefore requiring weaker additional measures. In this case, consumption drops immediately by only 10% (as compared to e.g. the 25% with generalized social distancing) and the overall death toll is limited to 0.18% of the total population.

The scenario presenting the second lowest need of shutdown is a generalized ("GEN") reduction by 20% of social contacts with milder policies on the young-young contacts. As young agents have the largest number of social interactions (see Figure 7), the loosening of social distancing for these agents implies a higher level of optimal enforced shutdown.

When young agents are allowed to interact in a socio-economic context the rise in infection calls for stricter economic shutdown measures generating the highest average output loss (24.9% losses with respect to the pre-epidemic).

In addition when social distancing is not enforced it is necessary to rise the economic shutdown level over time, as contagion increases more. However, the larger rise in the number of infected occurring in this scenario imply a dampening endogenous response of young agents' consumption acting in the same direction of the required rise in the level of economic shutdown and harming economic activity up to 40% downwards.

4.4 Containment policies comparison

To summarize what we learned from the analysis of the containment policies in the previous sections, we compare the different containment policies in terms of economic impact and death costs at the aggregate level. The comparison is done for the cases in which a vaccine is expected to be available in one year time from the epidemic outbreak.[27]

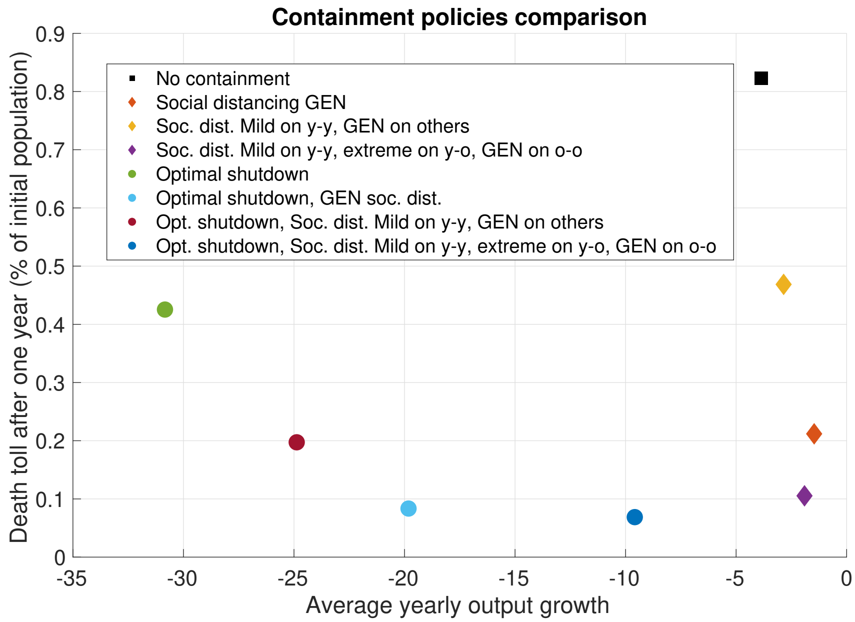

Figure 5: SIR-age macro: containment policies comparison

Note. The optimal economic shutdown parameter μ_c is computed in the case in which a vaccine is discovered in one year time from the outbreak of the epidemic for different social distancing scenarios. The scenario "generalized (GEN)" assumes that each social contact is diminished by 20% with respect to the baseline ("no distancing"). The scenario "mild on y-y, GEN on others" is the same as GEN but the social distancing among young individuals is 50% less severe than in GEN. The scenario "mild on y-y, extreme on y-o, GEN on o-o" is the same as the previous scenario but assumes that no contact is allowed between young and old individuals.

Figure 5 shows the deaths after one year (week 52) in percentage of the initial population (y-axis) and the average yearly output growth (x-axis) for different containment policies. The no containment scenario is characterized by the highest death toll of about 0.82% of the population and an economic loss of 3.86% of output as compared to the steady state.

In comparison to a no containment case, a generalized social distancing would attain a lower average yearly output loss, 1.47% and a lower death toll after one year of 0.21% of the population. A deviation from this generalized social distancing featuring young people entertaining more social interactions would result in higher deaths (0.47%) and higher output loss (2.85%) as compared to

[27]Notice that we assume that the social distancing measures are in place forever. This assumption is irrelevant as long as the course of the epidemic spans over a year after which, as we assume, a vaccine can become available. In reality, of course, there is much uncertainty on both the length of the epidemic and the effective availability of a vaccine.

generalized social distancing (however, it still represents an improvement in both dimensions as compared to no containment). A deviation from generalized social distancing via a reduction in older age groups contact with the younger is instead characterized by 1.9% output loss (worse than generalized social distancing, but better than mild social distancing for young) and 0.11% deaths as compared to initial population (better than both generalized and mild on young distancing).

An optimal shutdown measure is characterized by 0.43% deaths and 30.1% average yearly consumption losses, however in presence of social distancing the optimal shutdown will generate 19.8%, 24.9% and 9.6% average yearly output losses in the case of generalized social, milder on young and stricter on old social distancing respectively. The corresponding death tolls in the three cases are 0.08%, 0.2% and 0.07%. For any level of social distancing, the implied optimal economic shutdown generates small gains in terms of lives and large increases in terms of average output losses over one-year time.

5 Applying the model

Here we recalibrate the model to be consistent with the evolution of the epidemic as well as the economic shutdown and social distancing measures in Italy. We then investigate different possibilities for social distancing and economic shutdown for the progressive release of economic activity.

5.1 Re-calibrating the model

We consider the first week of 2020 as the *epidemic origin* (similarly to Favero et al. (2020)) and we recalibrate the model (as described in Appendix C) to match the number of effective deaths in week 10. According to our model it took two to three weeks to flatten the death curve after the imposed *lockdown* began (at the beginning of week 11).[28] Absent the lockdown measures in Italy,

[28] Data on COVID-19 infections and deaths are observed since week 9 (starting February 23). Week 11 (starting March 8) marks the beginning of the general shutdown of "inessential" economic activities and of enforced generalized social distancing: *Lockdown*. In week 19 (starting May 3) the government started by decree a post-lockdown period with milder containment measures while keeping a close monitoring of the total stock of infected (adjusting the measures of economic shutdown accordingly). May 3 is also when we stopped retrieving data on observed total deaths.

March and April would have seen a higher death toll according to our model, killing more than 0.4% of the initial population in a matter of 2 months.[29]

To be consistent with the death-toll observed in the data we assume that during the lockdown all contacts among individuals of any age group gets reduced to 46% of what would prevail in normal circumstances. To make our scenarios are also consistent with the official estimate of the Italian economic recession (real GDP) in the first quarter of 2020 (-5.3% in Q1 2020 with respect to the previous quarter, ISTAT (2020)) we set $\mu_c = 0.625$ in all periods of the lockdown phase.[30]

5.2 Epidemic management scenarios

We assume that the government post-lockdown sets the intensity of the economic shutdown (μ_c) in proportion to the observed stock of contemporaneous infected people:

$$\mu_{c,t} = \beta_\mu(I_{y,t} + I_{o,t}) \tag{5.1}$$

aiming at capturing the ready-to-intervene attitude of the government.[31]

Assuming $\beta_\mu = 9$ in all periods t in the post-lockdown phase, we obtain that the annual average growth rate of output in 2020 is in the range of -11.4% to -8.6% for the three main social distancing scenarios analyzed in Figure 6. This result compares, for example, with the estimate of -9.1% provided by the International Monetary Fund (2020). For the second quarter of 2020 compared with the first quarter of the same year the model generates an economic recession in the range of -18.8% to -12.2%. For comparison, European Commission (2020) projects the equivalent figure to be -13.6%.

We show the following possible scenarios for the post-lockdown phase:

(0) The reference case is a *laissez-faire* equilibrium (black lines in Figure 6) where individuals

[29] The official and effective data point to a total death-toll of 0.047% and 0.070% of the initial population respectively by the end of week 18 (April 26 to May 2).

[30] In particular, our model produces a recession in the first quarter of 2020 (compared to the previous quarter) of 5.3%, 5.2%, 4.9% corresponding to the blue, grey and green lines in Figure 8 in Appendix, respectively.

[31] Cf. "Phase 2, Press Conference of the Prime Minister, Giuseppe Conte", April 26, 2020, available at http://www.governo.it/node/14518.

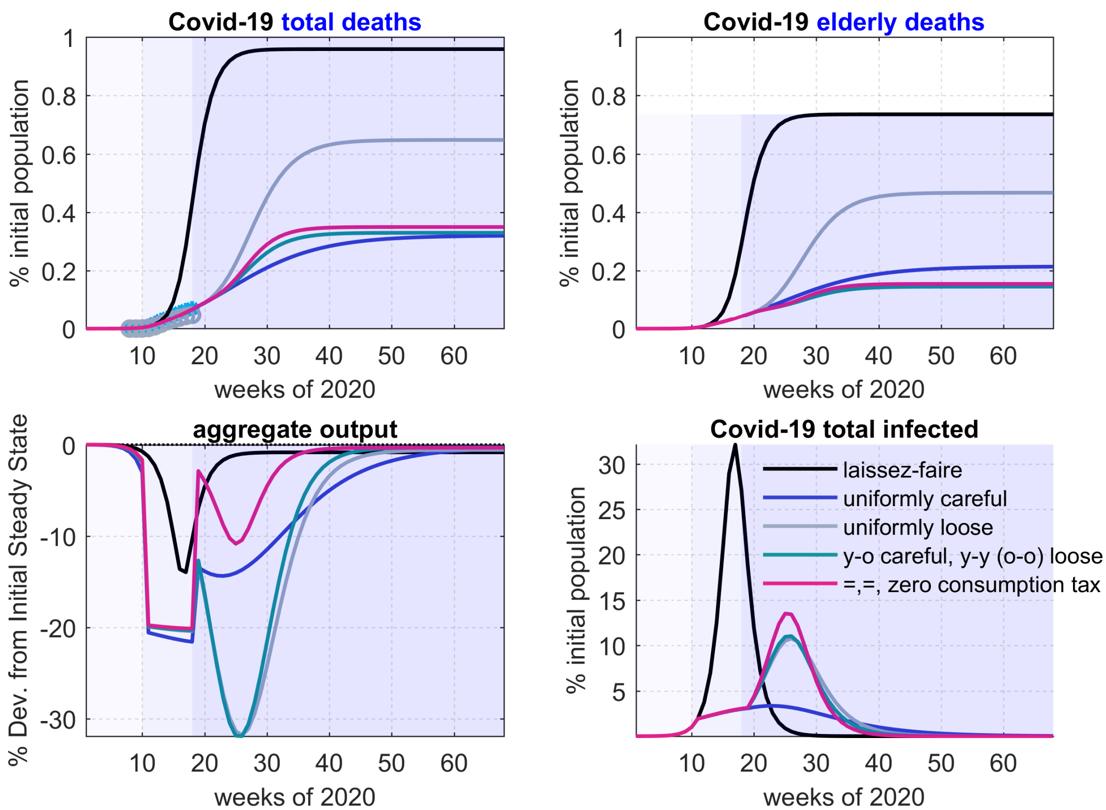

Figure 6: Different post-lockdown scenarios

maintain the same number of weekly contacts prevailing in the pre-epidemic times and the consumption tax is always set to zero. Our SIR-age macro model predicts that the economy would have faced a rapid recession with a trough at week 17 and an annual average growth rate of output in 2020 of -2.1% (vs -1.2% in the SIR-age model with no macroeconomic interactions). The cost of this milder recession is a high death-toll in the long-run of more than 0.9% of the initial population, i.e. more than 0.6 million lives.

The three post-lockdown social distancing policies are:

(1) *Uniformly careful* (blue lines in Figure 6), as careful as in the lockdown phase. Compared to the *laissez-faire* equilibrium, the final death-toll in this case is about 3 times smaller while the annual average growth rate of output in 2020 is about 4 times more negative. About two-thirds of the final death toll occur among the elderly.

(2) *Uniformly loose* (grey lines in Figure 6). The post lockdown social distancing is 50% looser than during the lockdown phase. Compared to scenario (1), this loosening doubles the final death toll, with more than two-thirds of deaths accounted by the elderly. The annual average growth rate of output in 2020 is -11.4%, worse than under scenario (1) (-8.6%) as the number of total infected increases slightly.

(3) *Careful with young-old, loose with young-young and old-old contacts* (green lines in Figure 6). We assume that the contacts among young and old individuals are two-thirds less than in the lockdown phase; the contacts among individuals of the same age are correspondingly two-thirds more. Under this scenario, the final death-toll is roughly the same one prevailing in scenario (1): there are more contacts, but the composition is changed and the elderly now account for a smaller share of the final death toll (44%). A new wave which is similar to the one under scenario (2) is present due to more cases, but these new infectious cases are not so lethal as they mostly occur among the young individuals. This results in an annual growth rate of output at -10.6%. While part of this bigger loss of output is due to the fact that more infected individuals decrease aggregate productivity and induce individuals (who discount a higher probability of being infected) to cut back on consumption and hours worked, the main reason is that we assume that the government sets the consumption tax proportionally to the number of contemporaneous infected. This is confirmed by the violet line in Figure 6 showing what would happen if the government was to set the consumption tax to zero in all post-lockdown periods. In this case the output loss in the second quarter (quarter on quarter) would be about 2.4 times smaller (compare with the green line).[32]

[32] Throughout the simulations we keep the number of contacts among the elderly ("old-old contacts") at the same level of looseness prevailing for contacts among the young. However, the old-old contact channel is not very important in the model at a macroeconomic level as the elderly are fewer in the macroeconomy (initially 16.7% of the population) and have comparably few contacts among themselves in normal times (see Figure 7).

6 Concluding remarks

Combining an epidemiological framework (Towers and Feng, 2012) into a macroeconomic one (Eichenbaum et al., 2020b), our model suggests that differentiating the policies to contain the COVID-19 crisis by age would be optimal. Compared to uniform social distancing and economic shutdown measures, age-targeted measures reduce the death toll while containing the output losses.

We calibrated the model for two age-groups: those aged 70 or more and the rest of the population. An extension would be to consider more age-groups that would allow to extend the set of policies as well as analysis of distributional policies and compensation policies. While highlighting the differences in impact of an economic shutdown and social distancing scenarios, this model can be extended to incorporating additional economic trade-off channels of social distancing especially relevant when analyzing phases of economic reopening.

Other important aspects that are not yet tackled in our analysis are the limits imposed by the hospitalization capacity and the possibility of other targeted polices focusing on the revealed health status of individuals. Aspects as the human costs of prolonged social distancing between young individuals and the elderly or time-varying developments of the COVID-19 transmissibility are not considered in our framework and one should remember to put into context the interpretation of our results.

Given the uncertainty surrounding many of the model parameters, we recognize that our quantitative results are most valuable in terms of a relative order of magnitude. In addition, together with the economic literature complementing our analysis, we contribute to the rising view that differentiating containment measures by age, acting more on social distancing of the elderly from the young is preferable to uniform and untargeted economic measures.

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Appendix

A First order conditions

The first order conditions are:

Susceptibles

$$c^s_{y,t} \quad : \quad \frac{1}{c^s_{y,t}} - (1+\mu_{c,t})\lambda^s_{y,t} + \eta\left(\pi^s_{y,1} z_{y,y} \frac{I_{y,t}}{f_y} c^i_{y,t} + \pi^s_{y,2} z_{y,o} \frac{I_{o,t}}{f_o} c^i_{o,t}\right)\lambda^\tau_{y,t} = 0 \tag{A.1}$$

$$n^s_{y,t} \quad : \quad -\theta n^s_{y,t} + w_t \lambda^s_{y,t} + \eta \pi^s_{y,4} z_{y,y} \frac{I_{y,t}}{f_y} n^i_{y,t} \lambda^\tau_{y,t} = 0 \tag{A.2}$$

$$\tau_{y,t} \quad : \quad \beta\left(U^i_{y,t+1} - U^s_{y,t+1}\right)(1-\delta_v) = \lambda^\tau_{y,t} \tag{A.3}$$

$$\lambda^s_{y,t} \quad : \quad (1+\mu_{c,t})c^s_{y,t} = w_t n^s_{y,t} + \Gamma_t \tag{A.4}$$

$$\lambda^s_{o,t} \quad : \quad c^s_{o,t} = \frac{\bar{P} + \Gamma_t}{1+\mu_{c,t}} \tag{A.5}$$

Infected

$$c^i_{y,t} \quad : \quad \frac{1}{c^i_{y,t}} - (1+\mu_{c,t})\lambda^i_{y,t} = 0 \tag{A.6}$$

$$n^i_{y,t} \quad : \quad -\theta n^i_{y,t} + \phi^i w_t \lambda^i_{y,t} = 0 \tag{A.7}$$

$$\lambda^i_{y,t} \quad : \quad (1+\mu_{c,t})c^i_{y,t} = w_t \phi^i n^i_{y,t} + \Gamma_t \tag{A.8}$$

$$\lambda^i_{o,t} \quad : \quad c^i_{o,t} = \frac{\bar{P} + \Gamma_t}{1+\mu_{c,t}} \tag{A.9}$$

Recovered

$$c^r_{y,t} \quad : \quad \frac{1}{c^r_{y,t}} - (1+\mu_{c,t})\lambda^r_{y,t} = 0 \tag{A.10}$$

$$n^r_{y,t} \quad : \quad -\theta n^r_{y,t} + w_t \lambda^r_{y,t} = 0 \tag{A.11}$$

$$\lambda^r_{y,t} \quad : \quad (1+\mu_{c,t})c^r_{y,t} = w_t n^r_{y,t} + \Gamma_t \tag{A.12}$$

$$\lambda^r_{o,t} \quad : \quad c^r_{o,t} = \frac{\bar{P} + \Gamma_t}{1+\mu_{c,t}} \tag{A.13}$$

B Computing the equilibrium

Following closely ERT, for a given sequence of containment rates $\{\mu_{c,t}\}_{t=0}^F$ for some final horizon F, guess sequences $\{n^s_{y,t}, n^i_{y,t}, n^r_{y,t}\}_{t=0}^F$ compute the sequence of the remaining unknown variables in each of the following equilibrium equations:

$$\lambda^r_{y,t} = \frac{\theta n^r_{y,t}}{A} \tag{B.1}$$

$$c^r_{y,t} = [(1+\mu_{c,t})\lambda^r_{y,t}]^{-1} \tag{B.2}$$

$$\Gamma_t = (1+\mu_{c,t})c^r_{y,t} - An^r_{y,t} \tag{B.3}$$

$$c^r_{o,t} = \frac{\bar{P}+\Gamma_t}{1+\mu_{c,t}} \tag{B.4}$$

$$u^r_{y,t} = \log c^r_{y,t} - \frac{\theta}{2}(n^r_{y,t})^2 \tag{B.5}$$

$$u^r_{o,t} = \log c^r_{o,t} \tag{B.6}$$

Iterate backwards from the post-epidemic steady-state values of $U^r_{y,t}, U^r_{o,t}$, $U^r_{y,F} = u^r_{y,t}/(1-\beta)$, $U^r_{o,F} = u^r_{o,t}/(1-\beta)$:

$$U^r_{y,t} = u^r_{y,t} + \beta U^r_{y,t+1} \tag{B.7}$$

$$U^r_{o,t} = u^r_{o,t} + \beta U^r_{o,t+1} \tag{B.8}$$

Calculate the sequence for remaining unknowns in the following equations:

$$\lambda^i_{y,t} = \frac{\theta n^i_{y,t}}{\phi^i A} \tag{B.9}$$

$$c^i_{y,t} = [(1+\mu_{c,t})\lambda^i_{y,t}]^{-1} \tag{B.10}$$

$$c^i_{o,t} = \frac{\bar{P}+\Gamma_t}{1+\mu_{c,t}} \tag{B.11}$$

$$u^i_{y,t} = \log c^i_{y,t} - \frac{\theta}{2}(n^i_{y,t})^2 \tag{B.12}$$

$$u^i_{o,t} = \log c^i_{o,t} \tag{B.13}$$

$$c^s_{y,t} = \frac{An^s_{y,t}+\Gamma_t}{1+\mu_{c,t}} \tag{B.14}$$

$$c^s_{o,t} = \frac{\bar{P}+\Gamma_t}{1+\mu_{c,t}} \tag{B.15}$$

$$u^s_{y,t} = \log c^s_{y,t} - \frac{\theta}{2}(n^s_{y,t})^2 \tag{B.16}$$

$$u^s_{o,t} = \log c^s_{o,t} \tag{B.17}$$

Given initial values $I_{o,0} = \varepsilon f_y$, $I_{y,0} = \varepsilon - I_{o,0}$, $S_{y,0} = (1-\varepsilon)f_y$, $S_{o,0} = 1-\varepsilon - S_{y,0}$, $N_{y,0} = f_y$, $N_{o,0} = f_o = 1 - f_y$, $N_0 = N_{y,0} + N_{o,0} = 1$, iterate forward the following equations for $t = 0, 1, ..., F-1$:

$$T_{y,t} = \eta S_{y,t}\left[\pi^s_{y,1} z_{y,y}\frac{I_{y,t}}{f_y}c^i_{y,t}c^s_{y,t} + \pi^s_{y,2} z_{y,o}\frac{I_{o,t}}{f_o}c^i_{o,t}c^s_{y,t} + \pi^s_{y,3} z_{y,y}\frac{I_{y,t}}{f_y}n^i_{y,t}n^s_{y,t} \right.$$
$$\left. + \pi^s_{y,4}\left(z_{y,y}\frac{I_{y,t}}{f_y} + z_{y,o}\frac{I_{o,t}}{f_o}\right)\right] \tag{B.18}$$

$$T_{o,t} = \eta S_{o,t}\left[\pi^s_{o,1} z_{o,o}\frac{I_{o,t}}{f_o}c^i_{o,t}c^s_{o,t} + \pi^s_{o,2} z_{o,y}\frac{I_{y,t}}{f_y}c^i_{y,t}c^s_{o,t} + \pi^s_{o,3}\left(z_{o,y}\frac{I_{y,t}}{f_y} + z_{o,o}\frac{I_{o,t}}{f_o}\right)\right] \tag{B.19}$$

$$S_{y,t+1} = S_{y,t} - T_{y,t} \tag{B.20}$$
$$S_{o,t+1} = S_{o,t} - T_{o,t} \tag{B.21}$$
$$I_{y,t+1} = I_{y,t} + T_{y,t} - (\pi_{y,r} + \pi_{y,d})I_{y,t} \tag{B.22}$$
$$I_{o,t+1} = I_{o,t} + T_{o,t} - (\pi_{o,r} + \pi_{o,d})I_{o,t} \tag{B.23}$$
$$R_{y,t+1} = R_{y,t} + \pi_{y,r}T_{y,t} \tag{B.24}$$
$$R_{o,t+1} = R_{o,t} + \pi_{o,r}T_{o,t} \tag{B.25}$$
$$D_{y,t+1} = D_{y,t} + \pi_{y,d}T_{y,t} \tag{B.26}$$
$$D_{o,t+1} = D_{o,t} + \pi_{o,d}T_{o,t} \tag{B.27}$$
$$N_{y,t} = S_{y,t} + I_{y,t} + R_{y,t} \tag{B.28}$$
$$N_{o,t} = S_{o,t} + I_{o,t} + R_{o,t} \tag{B.29}$$
$$N_{t+1} = N_{y,t+1} + N_{o,t+1} \tag{B.30}$$

Given the post-epidemic steady-state values of $U^i_{y,t}, U^i_{o,t}, U^s_{y,t}, U^s_{o,t}$:

$$\begin{aligned}
U^i_y &= \frac{u^i_y + \beta[\pi_{y,r}(1-\delta_c) + \delta_c]U^r_y}{1 - \beta(1 - \pi_{y,d} - \pi_{y,r})(1-\delta_c)} \\
U^i_o &= \frac{u^i_o + \beta[\pi_{o,r}(1-\delta_c) + \delta_c]U^r_o}{1 - \beta(1 - \pi_{o,d} - \pi_{o,r})(1-\delta_c)} \\
U^i_{y,F} &= (1-\delta_c)^F U^i_y + [1 - (1-\delta_c)^F]U^r_y \\
U^i_{o,F} &= (1-\delta_c)^F U^i_o + [1 - (1-\delta_c)^F]U^r_o \\
U^s_y &= \frac{u^s_y + \delta_v\beta U^r_y}{1 - \beta(1-\delta_v)} \\
U^s_o &= \frac{u^s_o + \delta_v\beta U^r_o}{1 - \beta(1-\delta_v)} \\
U^S_{y,F} &= (1-\delta_v)^F U^s_y + [1 - (1-\delta_v)^F]U^r_y \\
U^S_{o,F} &= (1-\delta_v)^F U^s_o + [1 - (1-\delta_v)^F]U^r_o
\end{aligned}$$

iterate backwards:

$$U^i_{y,t} = u^i_{y,t} + \beta\left[(1 - \pi_{y,d} - \pi_{y,r})U^i_{y,t+1} + \pi_{y,r}U^r_{y,t+1}\right](1-\delta_c) + \delta_c\beta U^r_{y,t+1} \tag{B.31}$$
$$U^i_{o,t} = u^i_{o,t} + \beta\left[(1 - \pi_{o,d} - \pi_{o,r})U^i_{o,t+1} + \pi_{o,r}U^r_{o,t+1}\right](1-\delta_c) + \delta_c\beta U^r_{o,t+1} \tag{B.32}$$

$$\tau_{y,t} = T_{y,t}/S_{y,t} \tag{B.33}$$
$$\tau_{o,t} = T_{o,t}/S_{o,t} \tag{B.34}$$

$$U^s_{y,t} = u^s_{y,t} + \beta\left[(1-\tau_{y,t})U^s_{y,t+1} + \tau_{y,t}U^i_{y,t+1}\right](1-\delta_v) + \delta_v\beta U^r_{y,t+1} \tag{B.35}$$
$$U^s_{o,t} = u^s_{o,t} + \beta\left[(1-\tau_{o,t})U^s_{o,t+1} + \tau_{o,t}U^i_{o,t+1}\right](1-\delta_v) + \delta_v\beta U^r_{o,t+1} \tag{B.36}$$

Calculate the sequence of remaining unknowns by the following equations:

$$\lambda^\tau_{y,t} = \beta\left(U^i_{y,t+1} - U^s_{y,t+1}\right)(1-\delta_v) \tag{B.37}$$
$$\lambda^s_{y,t} = \frac{\frac{1}{c^s_{y,t}} + \eta\left(\pi^s_{y,1}z_{y,y}\frac{I_{y,t}}{f_y}c^i_{y,t} + \pi^s_{y,2}z_{y,o}\frac{I_{o,t}}{f_o}c^i_{o,t}\right)\lambda^\tau_{y,t}}{(1+\mu_{c,t})} \tag{B.38}$$

Finally, given aggregate consumption:

$$C_t = c^s_{y,t}S_{y,t} + c^i_{y,t}I_{y,t} + c^r_{y,t}R_{y,t} + c^s_{o,t}S_{o,t} + c^i_{o,t}I_{o,t} + c^r_{o,t}R_{o,t} \tag{B.39}$$

use a gradient-based method to adjust the guesses $\left\{n^s_{y,t}, n^i_{y,t}, n^r_{y,t}\right\}_{t=0}^{F-1}$ so that the following three equations hold with arbitrary precision:

$$(1+\mu_{c,t})c^i_{y,t} - A\phi^i n^i_{y,t} - \Gamma_t = 0 \tag{B.40}$$
$$-\theta n^s_{y,t} + A\lambda^s_{y,t} + \eta\pi^s_{y,3}z_{y,y}\frac{I_{y,t}}{f_y}n^i_{y,t}\lambda^\tau_{y,t} = 0 \tag{B.41}$$
$$\mu_{c,t}C_t - \Gamma_t N_t = 0 \tag{B.42}$$

C Calibration details

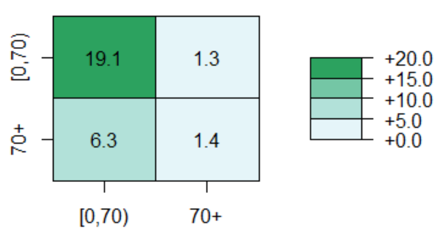

Figure 7: Daily contact matrix: 2 age-groups, Italy

Note. Elaboration on survey data from Mossong et al. (2017). The underlying matrix where each entry is multiplied by the relative demographic size of each age group is made symmetric employing the same demographic shares employed in the model, namely $f_y = 0.825$, $f_o = 1 - f_y$.

C.1 Calibration to the Italian case

The official data on COVID-19 deaths, published by the *Protezione Civile* of the Italian government, are available from February 24, 2020.[33] As noted by Galeotti et al. (2020), the official numbers might under report the actual death-toll of the virus in the Italian population. We build a counterfactual of the 2020 total deaths in absence of COVID-19 based on the trend in the preceding five years and obtain an estimate of the COVID-19 effective number of deaths on a weekly basis.[34] Throughout the calibration we aim at targeting these epidemiological data and we will consider as goodness of fit whether our model's simulation series will lay in-between those two data curves in the lockdown phase.

The first step of the calibration we did is to set the value of R_0, the basic reproduction number, in the purely epidemiological ("SIR-age") model to a value consistent with the Italian case. While there is a notorious uncertainty surrounding this statistic, in an early study based on Lombardy (the Northern Italian region most hit by COVID-19) Cereda et al. (2020) report a value of 3.1 (95% CI, 2.9 to 3.2), which is the value we assume. Applying equation 3.1, this value of the reproduction number implies $\eta = 0.01129$. We depart from ERT's strategy of employing COVID-19 mortality rates on reported cases in South Korea, turning instead to the estimates of the infection fatality rate (IFR) by age for Italy provided by Rinaldi and Paradisi (2020). For the two age classes of our interest, their central estimates – once interacted with the Italian demographic shares – give the following probabilities of dying upon infection:

$$\frac{\pi_{y,d}}{\pi_{y,r} + \pi_{y,d}} = 0.00278 \qquad\qquad \frac{\pi_{o,d}}{\pi_{o,r} + \pi_{o,d}} = 0.06274$$

Given their reported estimates, we consider that the elderly are those aged 71 or more (71+) while the rest of the population figures as young. The initial demographic shares, using data from the United Nations World Population Prospects 2019, are $f_y = 0.833$, $f_o = 1 - f_y = 0.167$.

Furthermore, we assume that it takes on average 14 days to either recover or die from the infection (cf. Eichenbaum et al. (2020a)). Hence, given that our model is calibrated at the weekly frequency, we set $\pi_{y,r} + \pi_{y,d} = \pi_{y,r} + \pi_{y,d} = \gamma = 7/14$.

Given the same main parameter values of section 3.1, we run the SIR-age model finding the following limiting values:

$$\lim_{t\to\infty} \{R_{y,t} + D_{y,t}\} = 0.8189 \qquad\qquad \lim_{t\to\infty} \{R_{o,t} + D_{o,t}\} = 0.1214$$

that we used as new values in the SIR-age macro model to calibrate the $\pi_{\cdot,\cdot}$. To do so we had to choose the size of the initial shock to the total amount of infected. We found that a value of $\varepsilon = 0.0000225$ gave a number of total deaths in the SIR-age macro model consistent with the value of total effective deaths in the first and second weeks observed in the data (see Figure 8).

[33] We used data until May 3, 2020, retrieved from this data repository `https://github.com/pcm-dpc/COVID-19/tree/master/dati-andamento-nazionale` which is updated on a daily basis.

[34] The methodology exploits the data released by the Italian Office for National Statistics (ISTAT), available at `https://www.istat.it/it/archivio/240401` - the specific dataset used is "Dataset analitico con i decessi giornalieri", which comprises the daily total deaths (by any cause) in 2020 until April 4 in the 1689 Italian municipalities most affected by COVID-19. Contrary to Galeotti et al. (2020), we also impute a value to the effective number of deaths due to COVID-19 beyond April 4 (until May 3). We do so applying the last observed 'bias factor' (the ratio of the stock of COVID-19 effective deaths over the official ones) which is 1.48. We find that this statistic was very high, at 25, in the first week of official data release (February 23 to 29) and then decreased progressively: 8.6, 3.9, 2.45, 1.86 and 1.48 (in the last week from March 29 to April 4).

This amounts to assume that in the first week of 2020 about 1360 people were infected.

In the SIR-age macro model we always set the vaccination probability δ_v to $1/52$ which implies that it takes on average 52 weeks (i.e. 1 year) for the vaccine to become available.[35]

Figure 8 shows both the effective and the official data series for the total deaths caused by COVID-19 in Italy in the weeks corresponding to the period between February 24 and May 3, 2020, as percent of the initial population (according to United Nations World Population Prospects 2019 the Italian population stood at 60.55 million people in 2019).

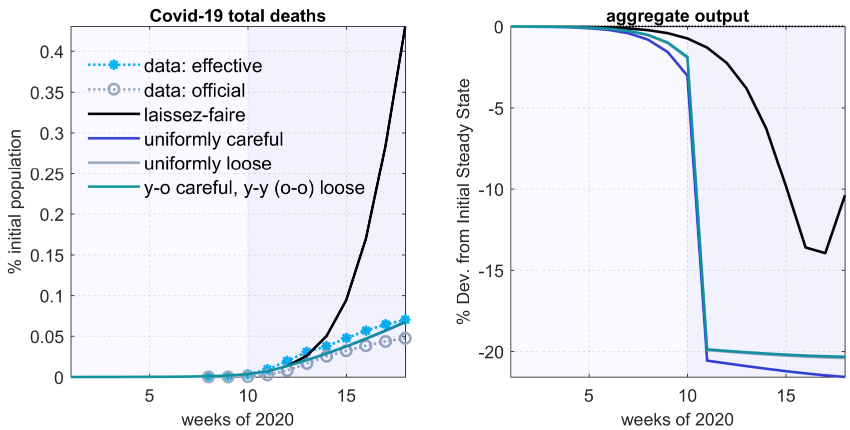

Figure 8: The lockdown phase calibration: data vs model

D Additional figures

[35] We do so because otherwise it would be hard to justify such a strict observed lockdown in Phase 1 (if not for other reasons such as the overrunning of the hospitalization capacity which is a factor that we do not consider in the current analysis). As also found by ERT, and as we have documented in the previous section, this assumption leaves essentially unaltered the resulting *laissez-faire* equilibrium outcomes. It matters, however, for the implied optimal economic shutdown.

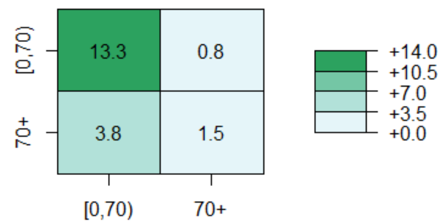

Figure 9: Daily contact Matrix: **2 age-groups, full sample**

Note. Elaboration on survey data from Mossong et al. (2017). Full sample comprises: Italy, Germany, Luxembourg, Netherlands, Poland, United Kingdom, Finland, Belgium. The underlying matrix (where each entry is multiplied by the relative demographic size of each age group) is made symmetric employing the same demographic shares employed in the main text, namely $f_y = 0.825$, $f_o = 1 - f_y$.

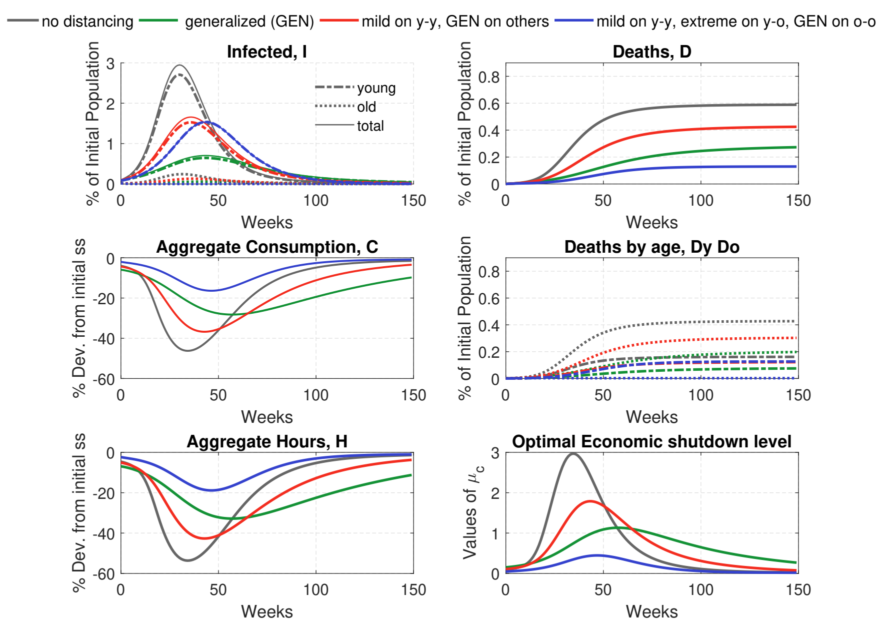

Figure 10: SIR-age macro model: Optimal shutdown with social distancing

Note. The optimal economic shutdown parameter μ_c is computed in the case in which a vaccine is never discovered for different social distancing scenarios. The scenario "generalized (GEN)" assumes that each social contact is diminished by 20% with respect to the baseline ("no distancing"). The scenario "mild on y-y, GEN on others" is the same as GEN but the social distancing among young individuals is 50% less severe than in GEN. The scenario "mild on y-y, extreme on y-o, GEN on o-o" is the same as the previous scenario but assumes that no contact is allowed between young and old individuals.